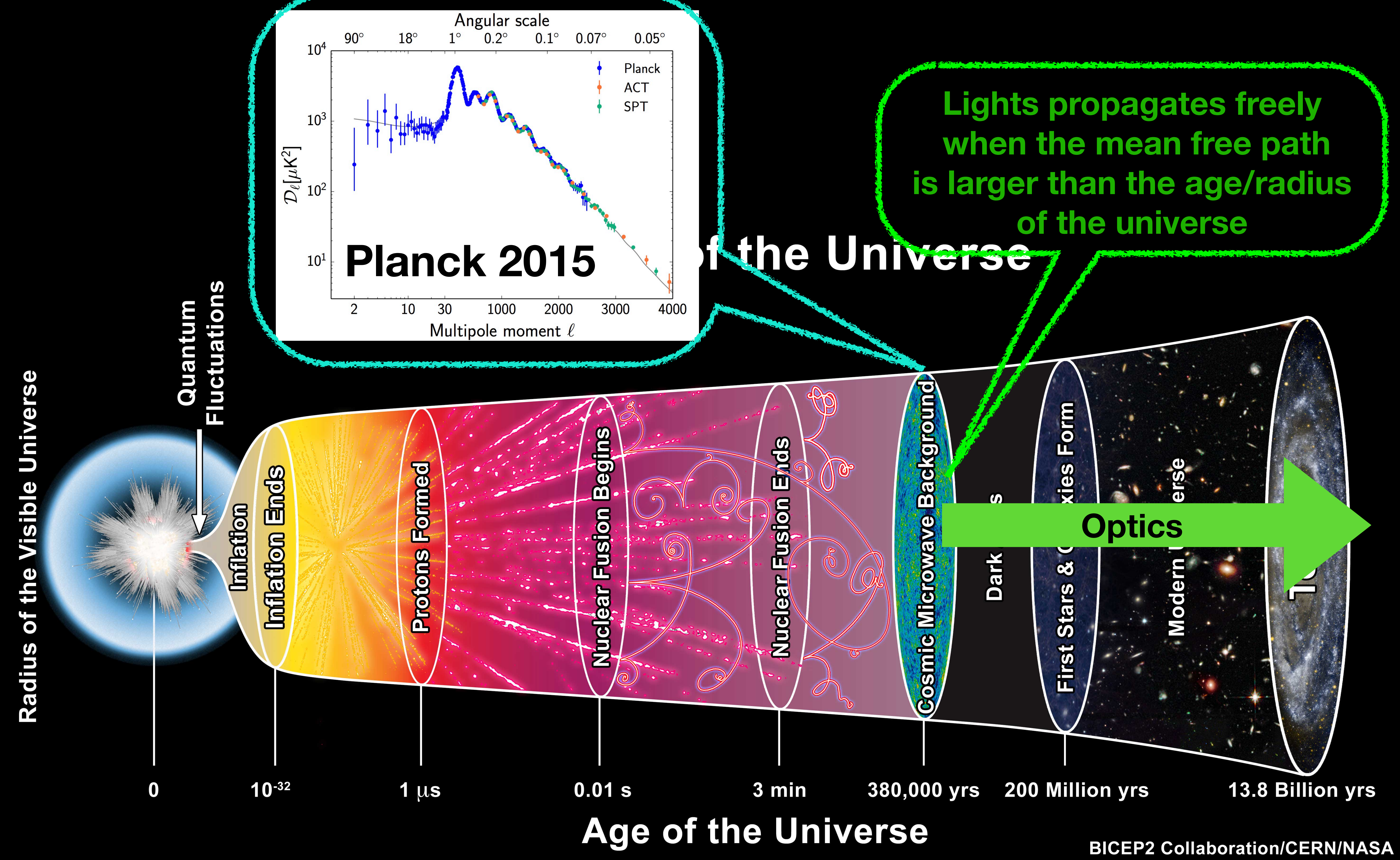
Some Properties of Stochastic Gravitational Wave Background

Shi Pi (皮石)
Kavli IPMU, the University of Tokyo/
Institute for Theoretical Physics, CAS

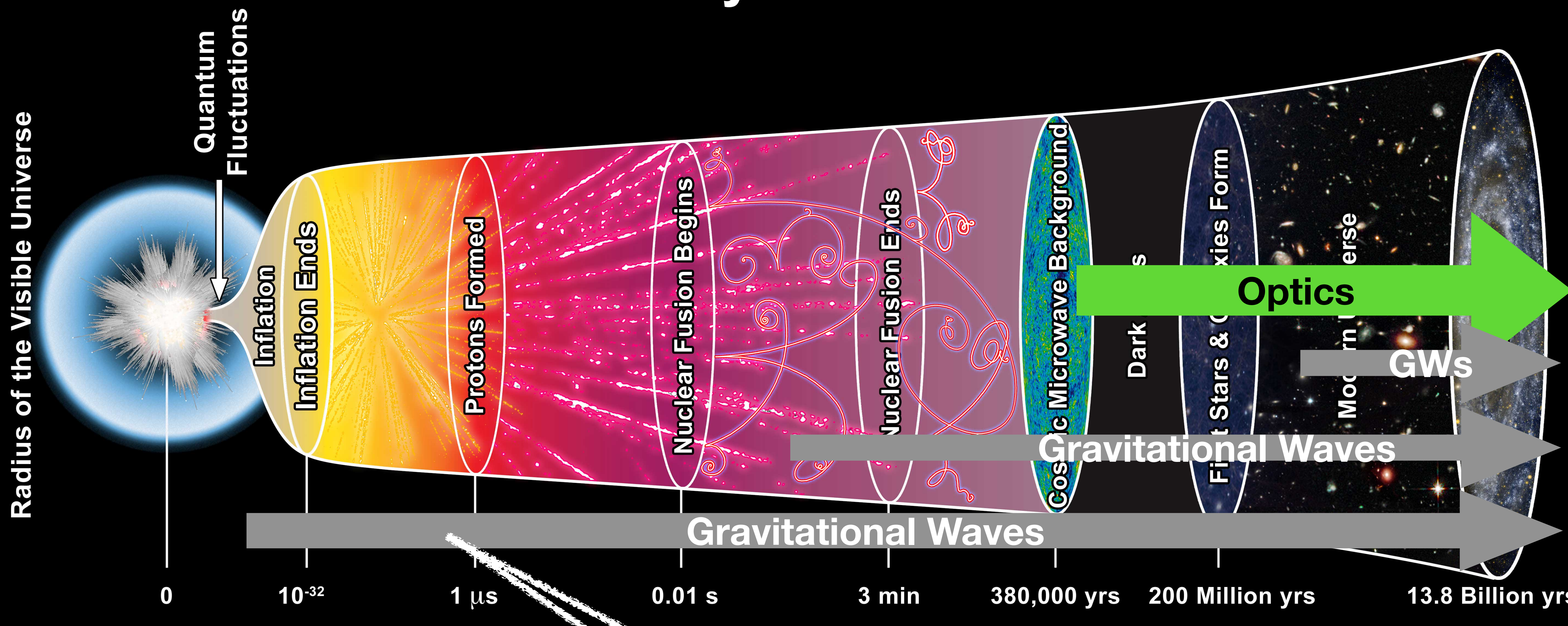
The Interdisciplinary Center for Theoretical Study, USTC, 2020.7.24

Content

- Introduction
- Primordial perturbation and PBHs
- Induced GWs, and their relation to PBHs
- Some properties for IGW: scaling and peak
- Summary


Angular scale
90° 18° 1° 0.2° 0.1° 0.07° 0.05°
Planck
ACT
SPT
$\mathcal{D}_\ell [\mu K^2]$
10^4 10^3 10^2 10^1
2 10 30 1000 2000 3000 4000
Multipole moment ℓ
Planck 2015
Lights propagates freely
when the mean free path
is larger than the age/radius
of the universe
Radius of the Visible Universe
Quantum Fluctuations
Inflation
Inflation Ends
Protons Formed
Nuclear Fusion Begins
Nuclear Fusion Ends
Cosmic Microwave Background
Optics
0
10^{-32}
1 μs
0.01 s
3 min
380,000 yrs
200 Million yrs
13.8 Billion yrs
Age of the Universe
BICEP2 Collaboration/CERN/NASA

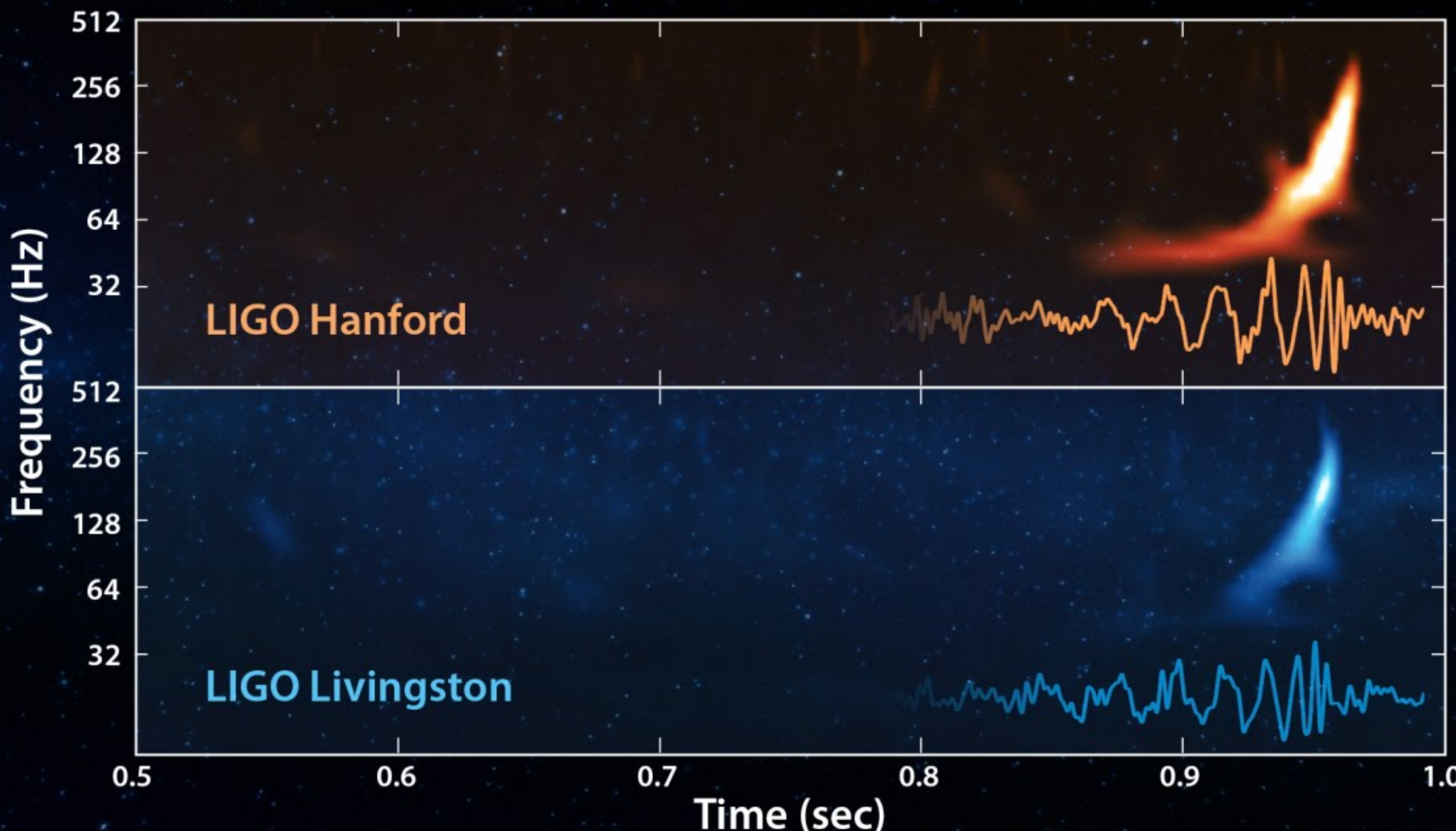
History of the Universe



BICEP2 Collaboration/CERN/NASA

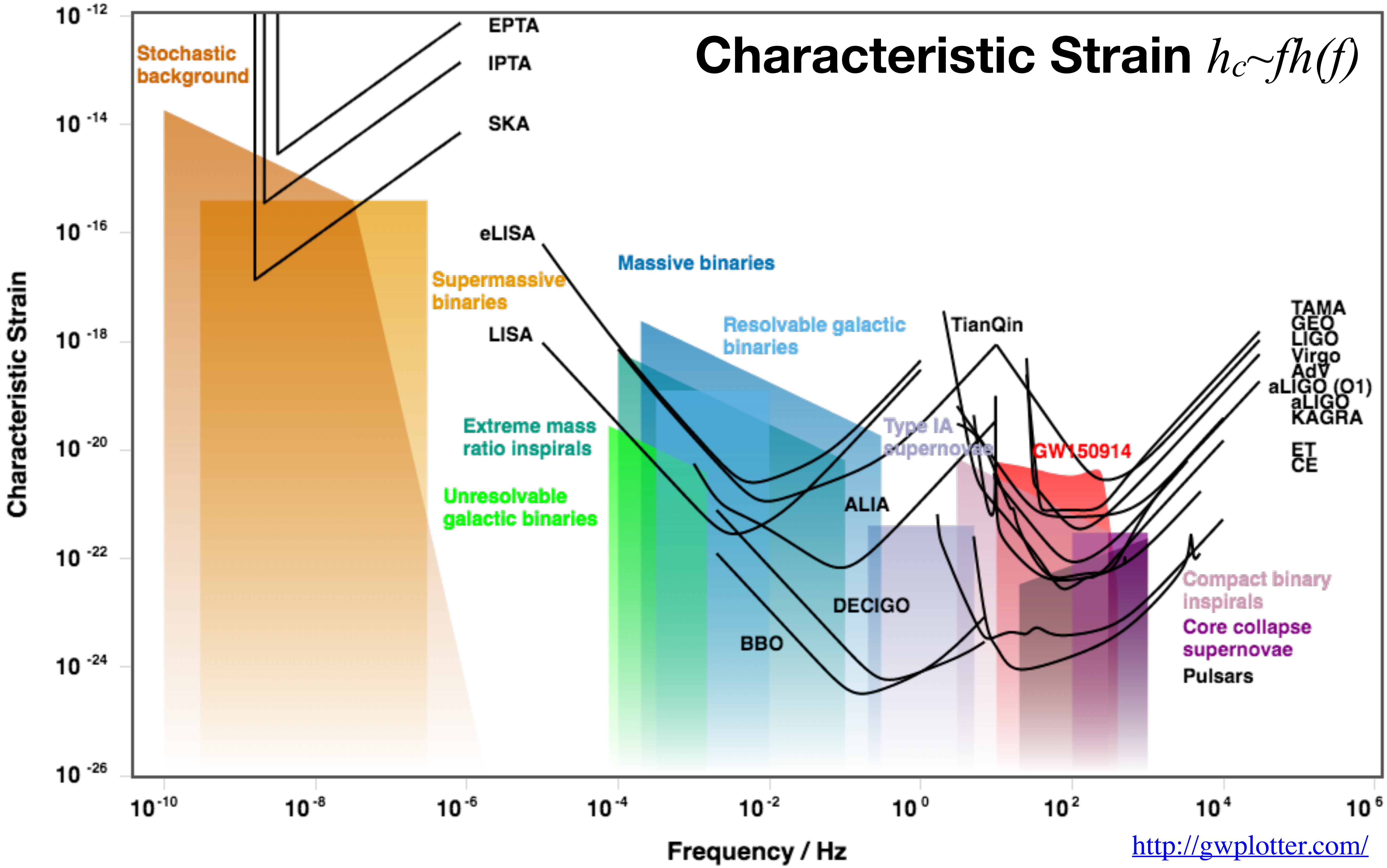
GWs propagates freely all the time!

LIGO detection of GWs



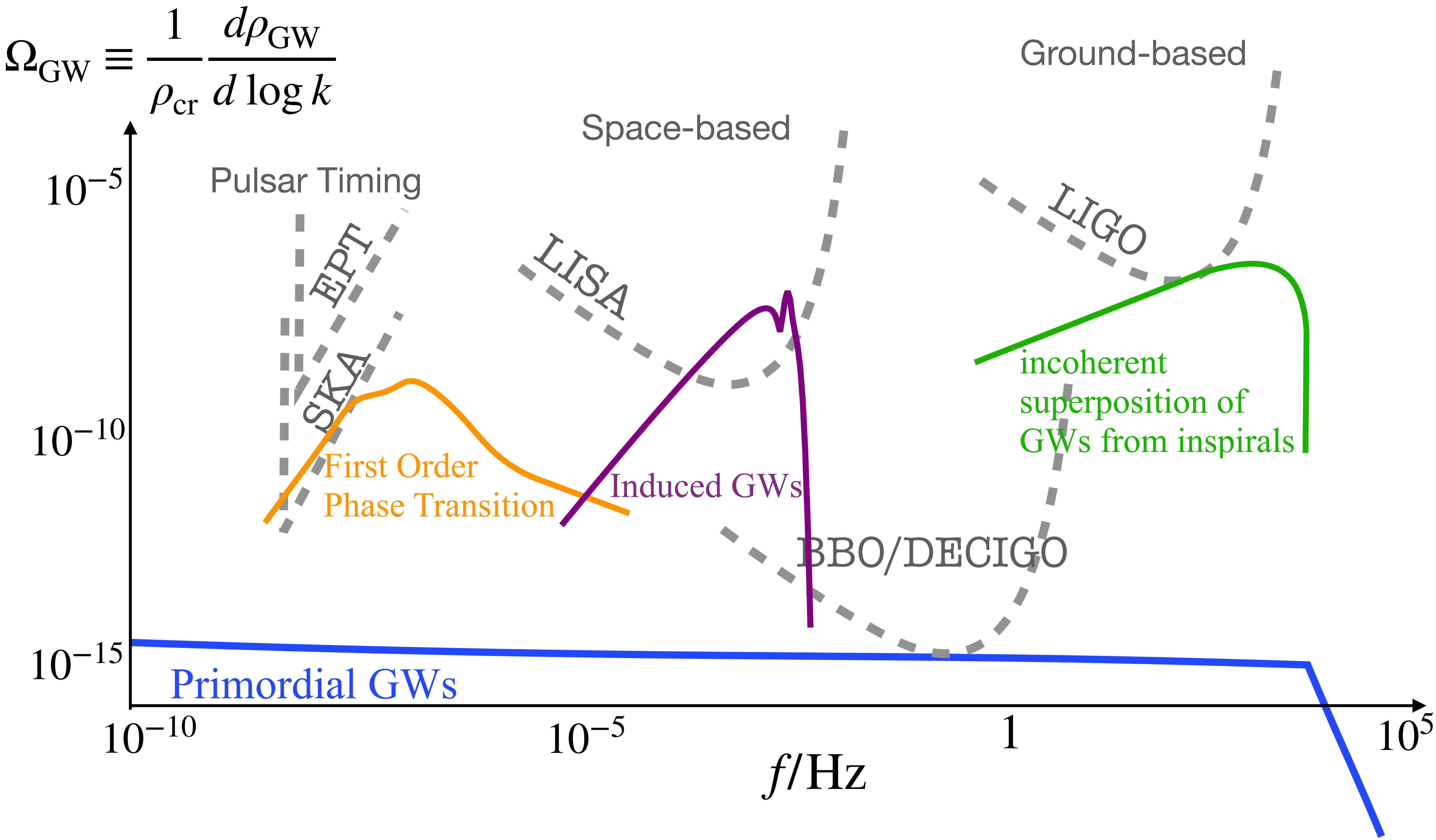
ligo.org

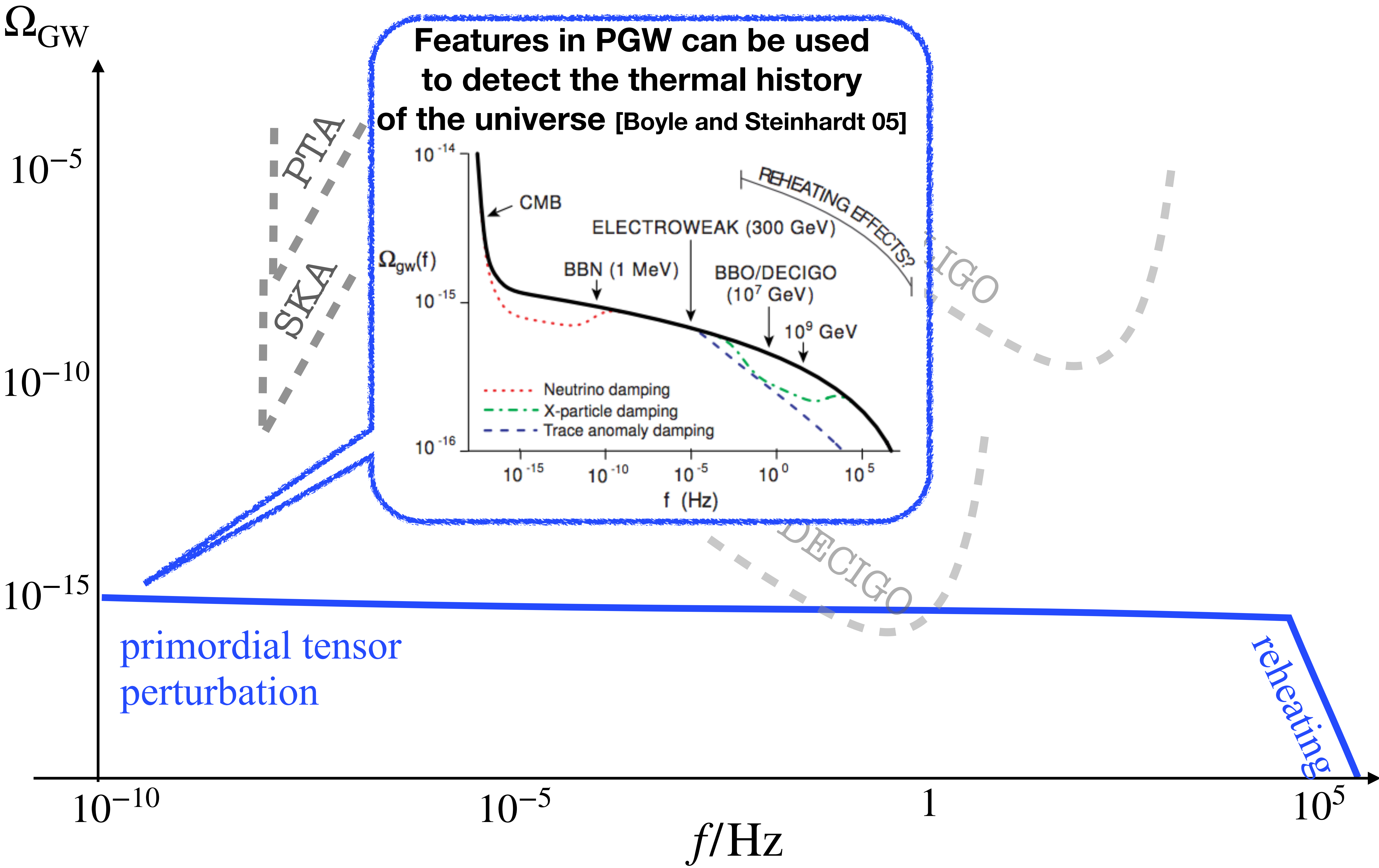
Future GW detectors



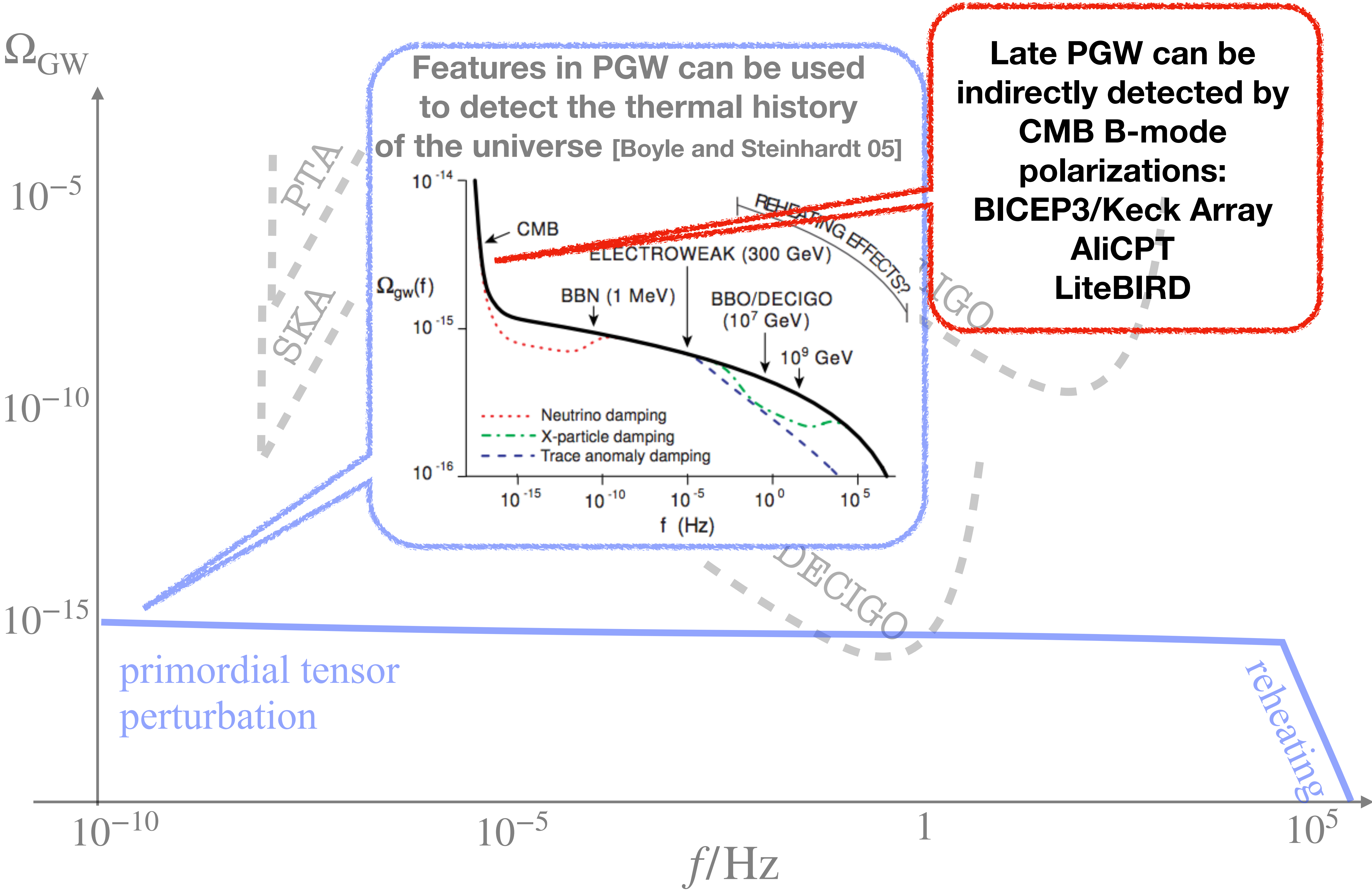
http://gwplotter.com/

Possible SGWB Sources

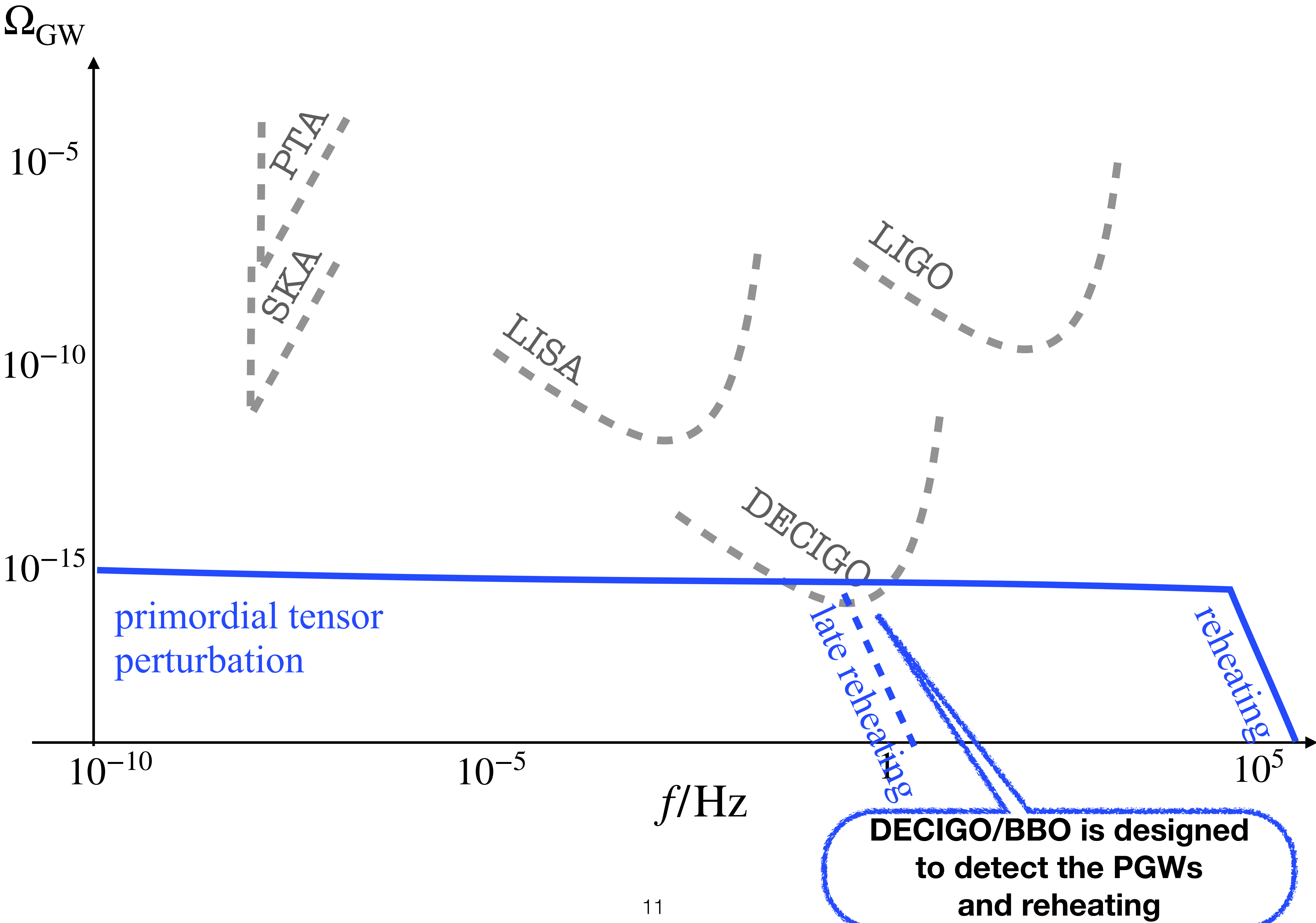




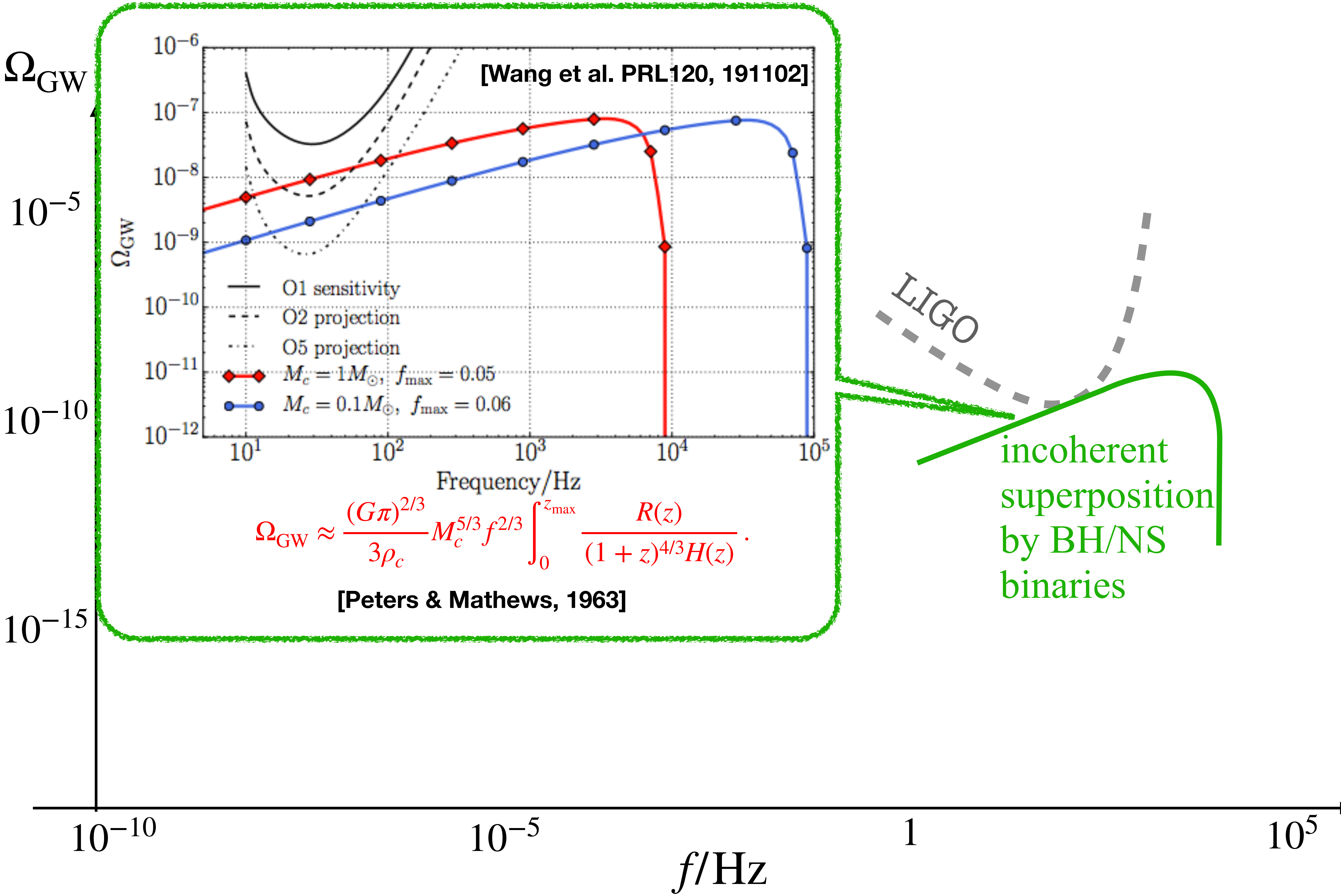
$\Omega_{\rm GW}$
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LIGO
DECIGO
Features in PGW can be used to detect the thermal history of the universe [Boyle and Steinhardt 05]
10^{-14}
10^{-15}
10^{-16}
$\Omega_{gw}(f)$
CMB
BBN (1 MeV)
ELECTROWEAK (300 GeV)
BBO/DECIGO (10^7 GeV)
10^9 GeV
REHEATING EFFECTS?
Neutrino damping
X-particle damping
Trace anomaly damping
10^{-15} 10^{-10} 10^{-5} 10^0 10^5
f (Hz)
primordial tensor perturbation
reheating
10^{-10}
10^{-5}
1
10^5
f/Hz



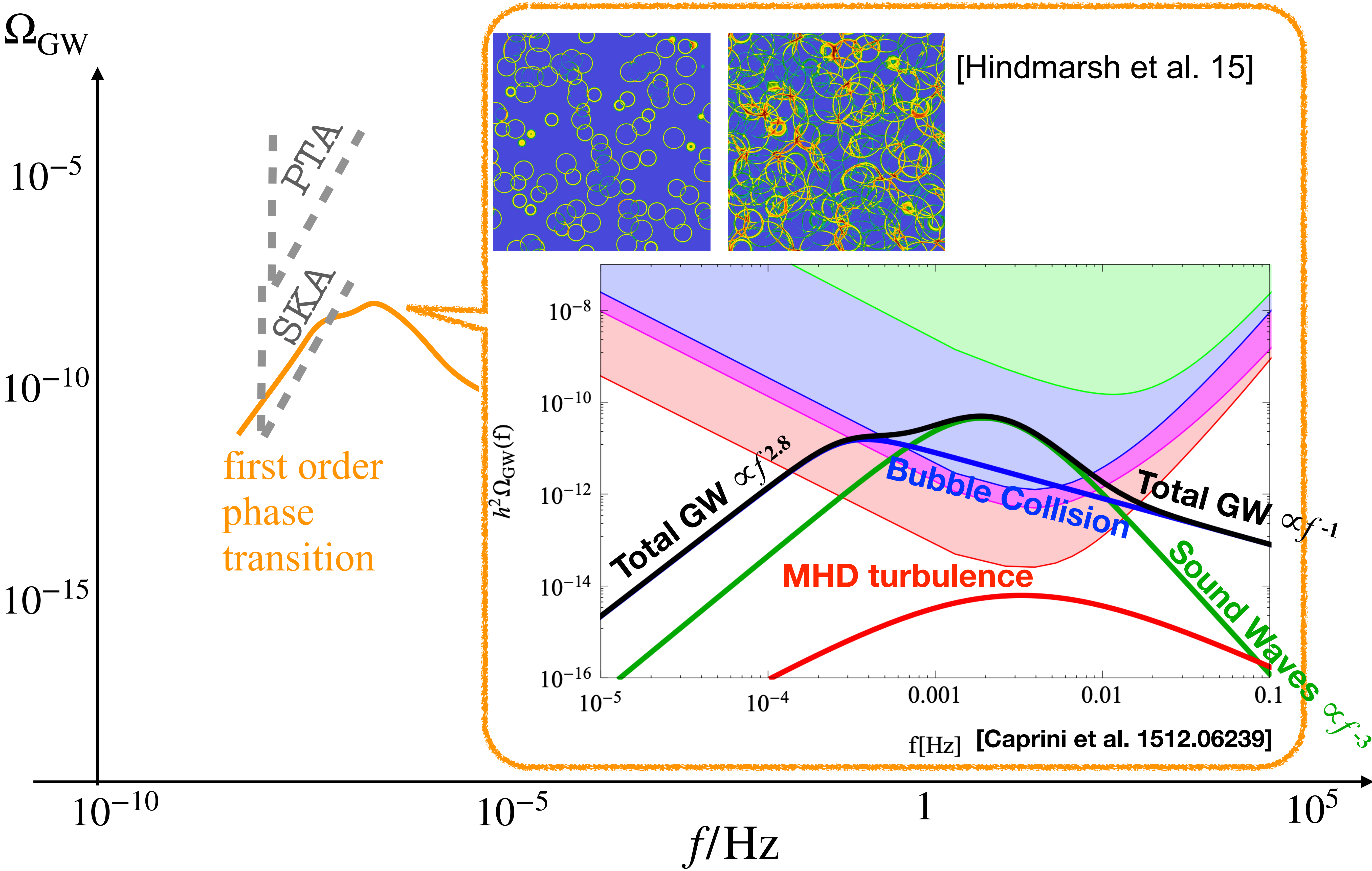
Features in PGW can be used to detect the thermal history of the universe [Boyle and Steinhardt 05]
Late PGW can be indirectly detected by CMB B-mode polarizations:
BICEP3/Keck Array
AliCPT
LiteBIRD
Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LIGO
DECIGO
primordial tensor perturbation
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz
$\Omega_{gw}(f)$
CMB
ELECTROWEAK (300 GeV)
BBN (1 MeV)
BBO/DECIGO (10^7 GeV)
10^9 GeV
REHEATING EFFECTS?
Neutrino damping
X-particle damping
Trace anomaly damping
f (Hz)



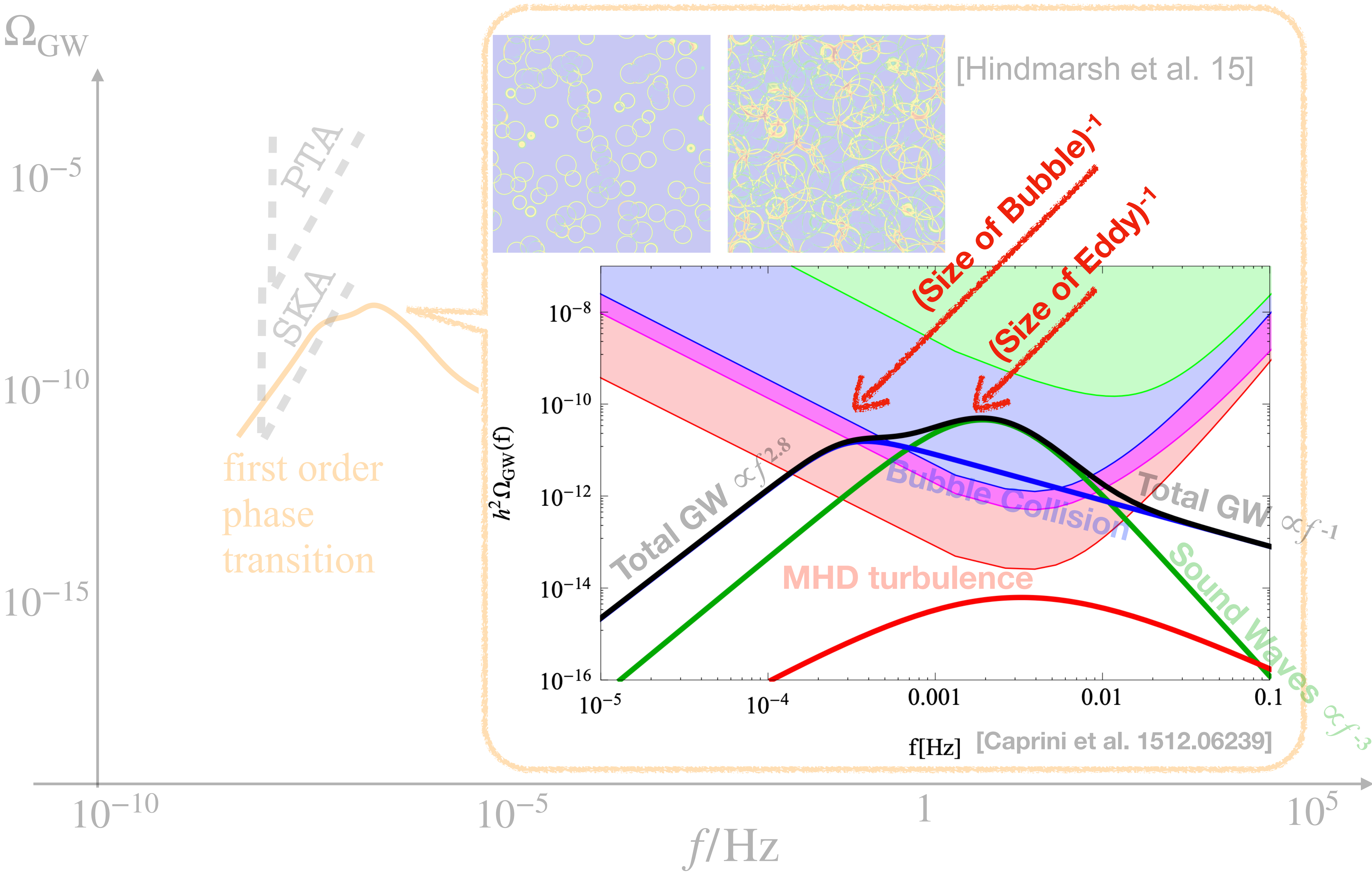
$\Omega_{\rm GW}$
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
LIGO
DECIGO
primordial tensor perturbation
late reheating
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz
DECIGO/BBO is designed to detect the PGWs and reheating



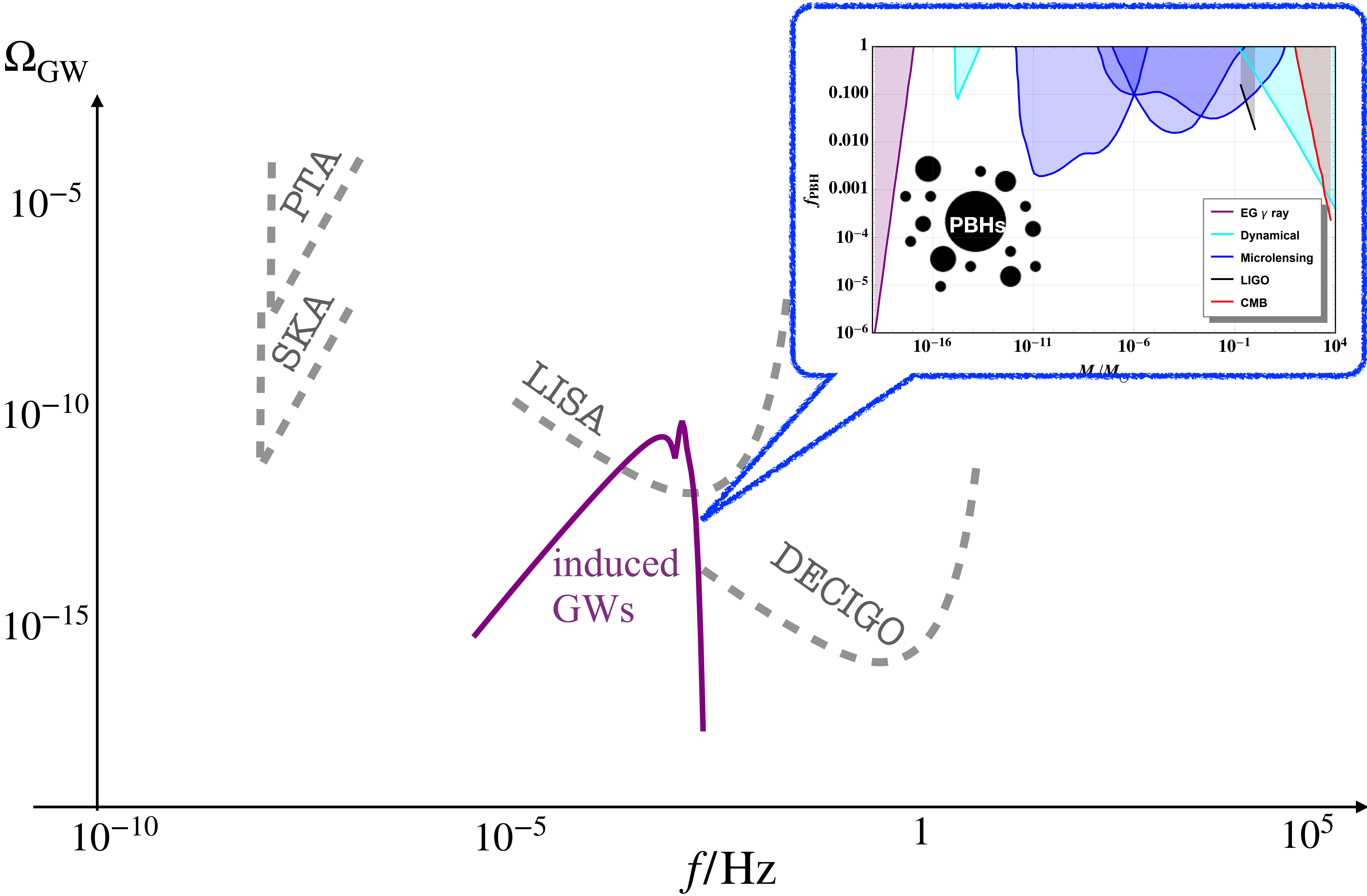
Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
10^{-10}
10^{-5}
1
10^5
f/Hz
[Wang et al. PRL120, 191102]
O1 sensitivity
O2 projection
O5 projection
$M_c = 1M_\odot,\ f_{max} = 0.05$
$M_c = 0.1M_\odot,\ f_{max} = 0.06$
Frequency/Hz
$\Omega_{GW} \approx \frac{(G\pi)^{2/3}}{3\rho_c} M_c^{5/3} f^{2/3} \int_0^{z_{max}} \frac{R(z)}{(1+z)^{4/3} H(z)}.$
[Peters & Mathews, 1963]
LIGO
incoherent superposition by BH/NS binaries



Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
first order
phase
transition
[Hindmarsh et al. 15]
$h^2\Omega_{GW}(f)$
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
Total GW $\propto f^{2.8}$
Bubble Collision
Total GW $\propto f^{-1}$
MHD turbulence
Sound Waves $\propto f^{-3}$
10^{-5}
10^{-4}
0.001
0.01
0.1
f[Hz]
[Caprini et al. 1512.06239]
10^{-10}
10^{-5}
1
10^{5}
f/Hz



$\Omega_{\rm GW}$
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
first order phase transition
[Hindmarsh et al. 15]
(Size of Bubble)$^{-1}$
(Size of Eddy)$^{-1}$
$h^2\Omega_{\rm GW}(f)$
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
Total GW $\propto f^{2.8}$
Bubble Collision
Total GW $\propto f^{-1}$
MHD turbulence
Sound Waves $\propto f^{-3}$
10^{-5}
10^{-4}
0.001
0.01
0.1
f[Hz]
[Caprini et al. 1512.06239]
10^{-10}
10^{-5}
1
10^{5}
f/Hz

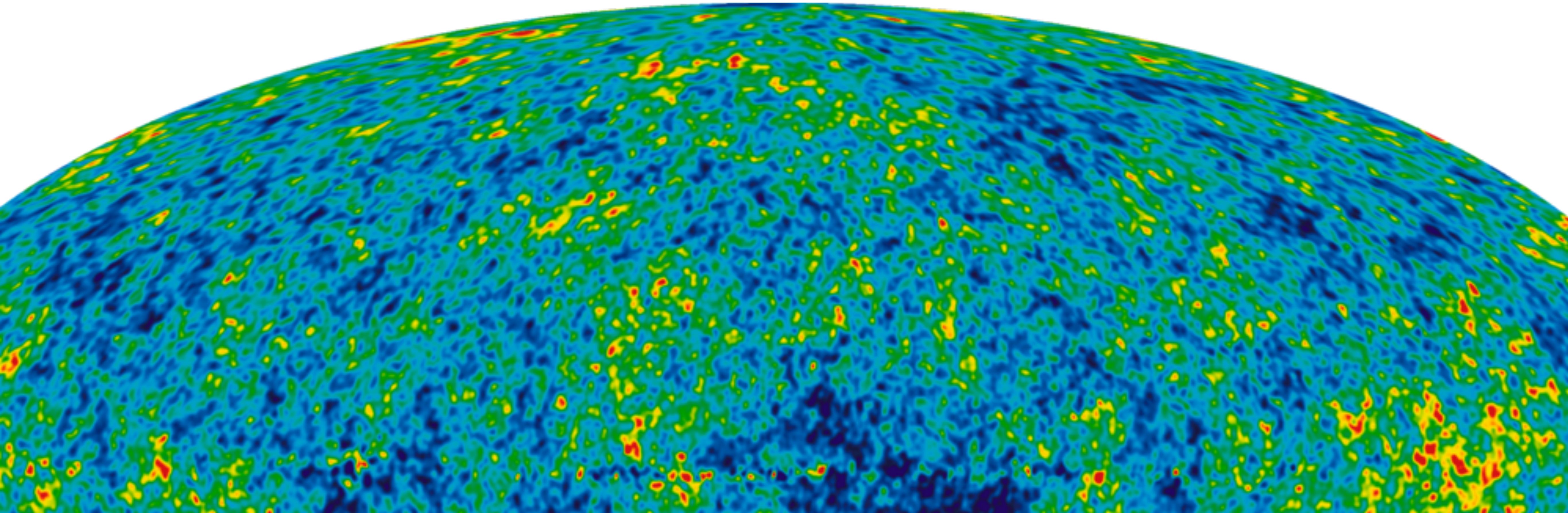


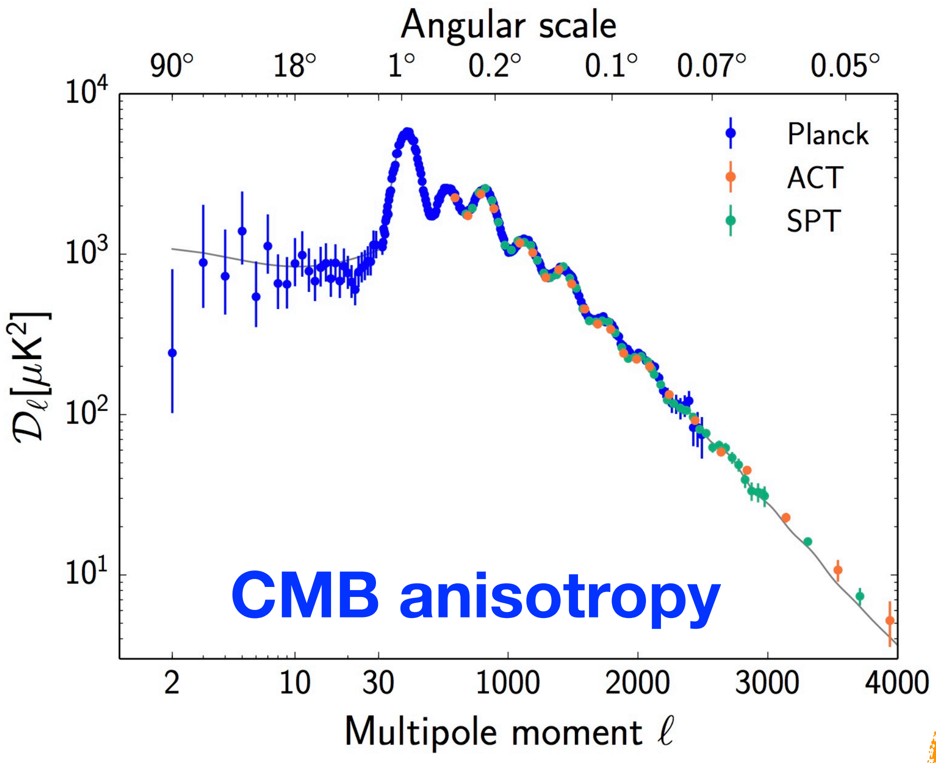
Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
induced GWs
DECIGO
10^{-10}
10^{-5}
1
10^{5}
f/Hz
f_{PBH}
PBHs
EG γ ray
Dynamical
Microlensing
LIGO
CMB

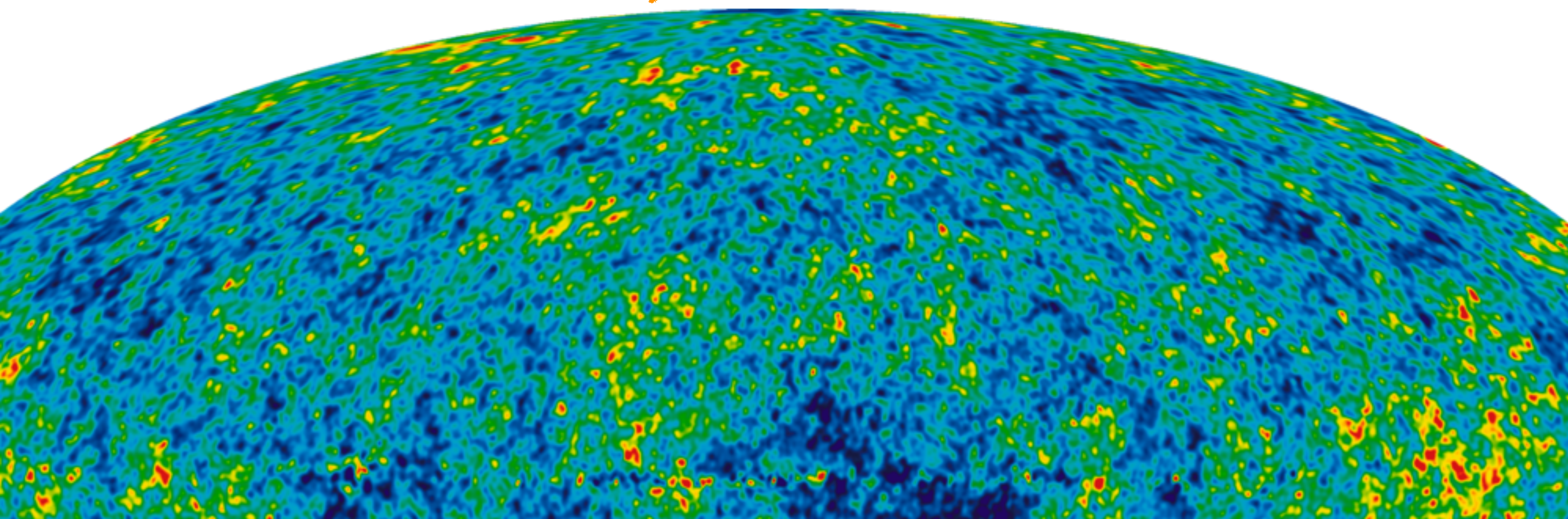
Content

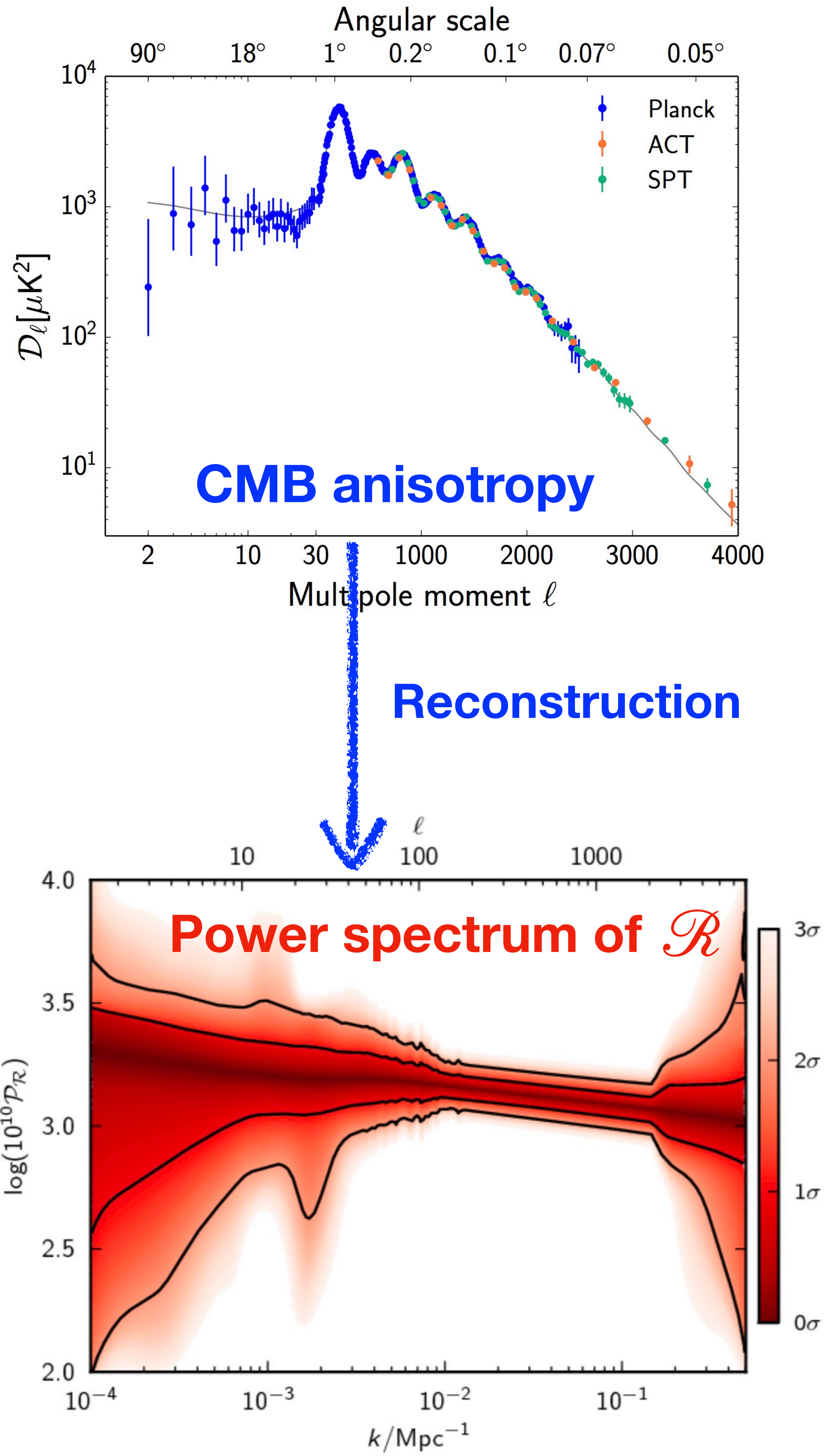
- Introduction
- Primordial perturbation and PBHs
- Induced GWs, and their relation to PBHs
- Some properties for IGW: scaling and peak
- Summary

Introduction to Induced GW (and PBH)

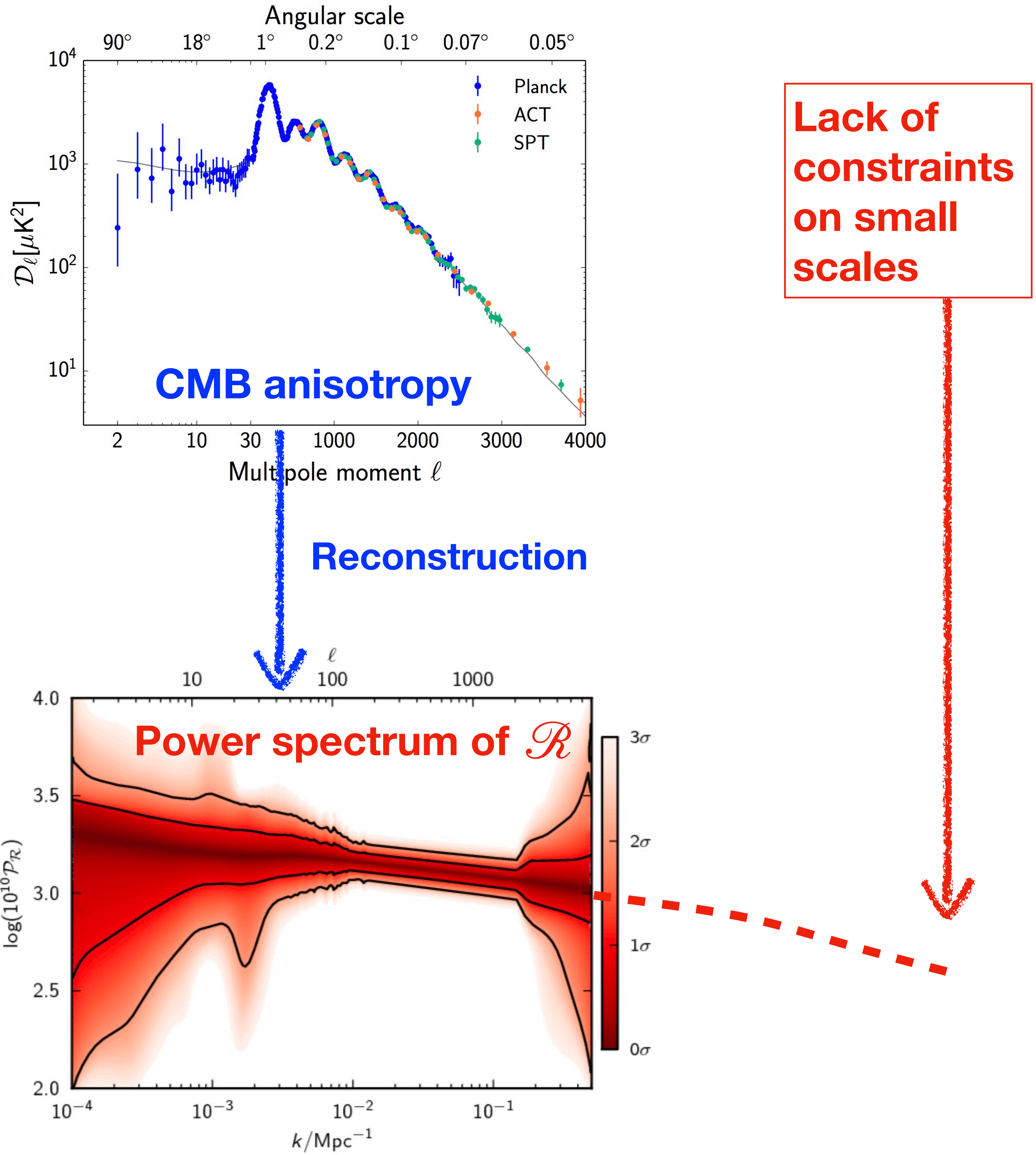



Angular scale
90°
18°
1°
0.2°
0.1°
0.07°
0.05°
Planck
ACT
SPT
CMB anisotropy
$\mathcal{D}_\ell[\mu K^2]$
2
10
30
1000
2000
3000
4000
Multipole moment ℓ

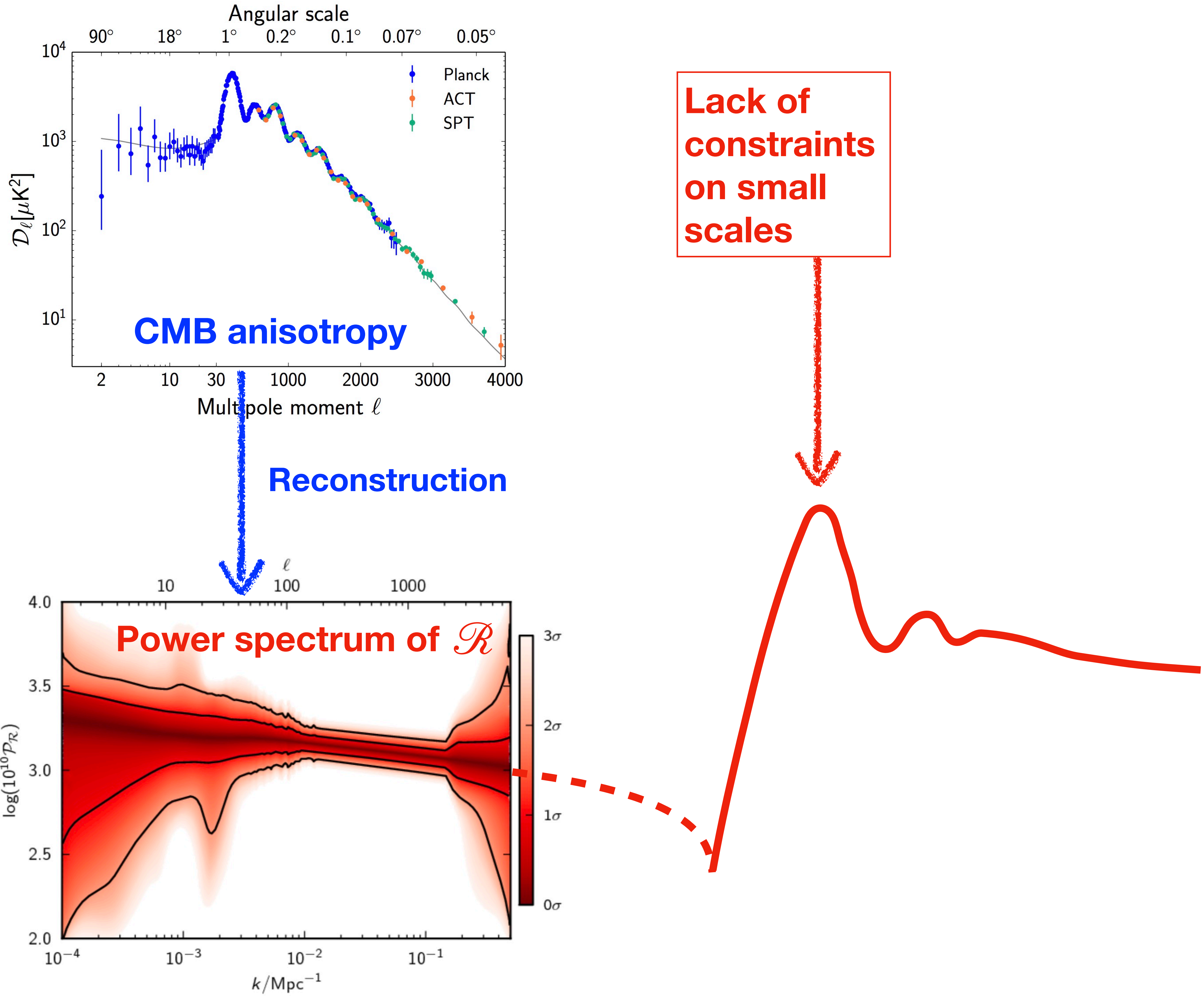


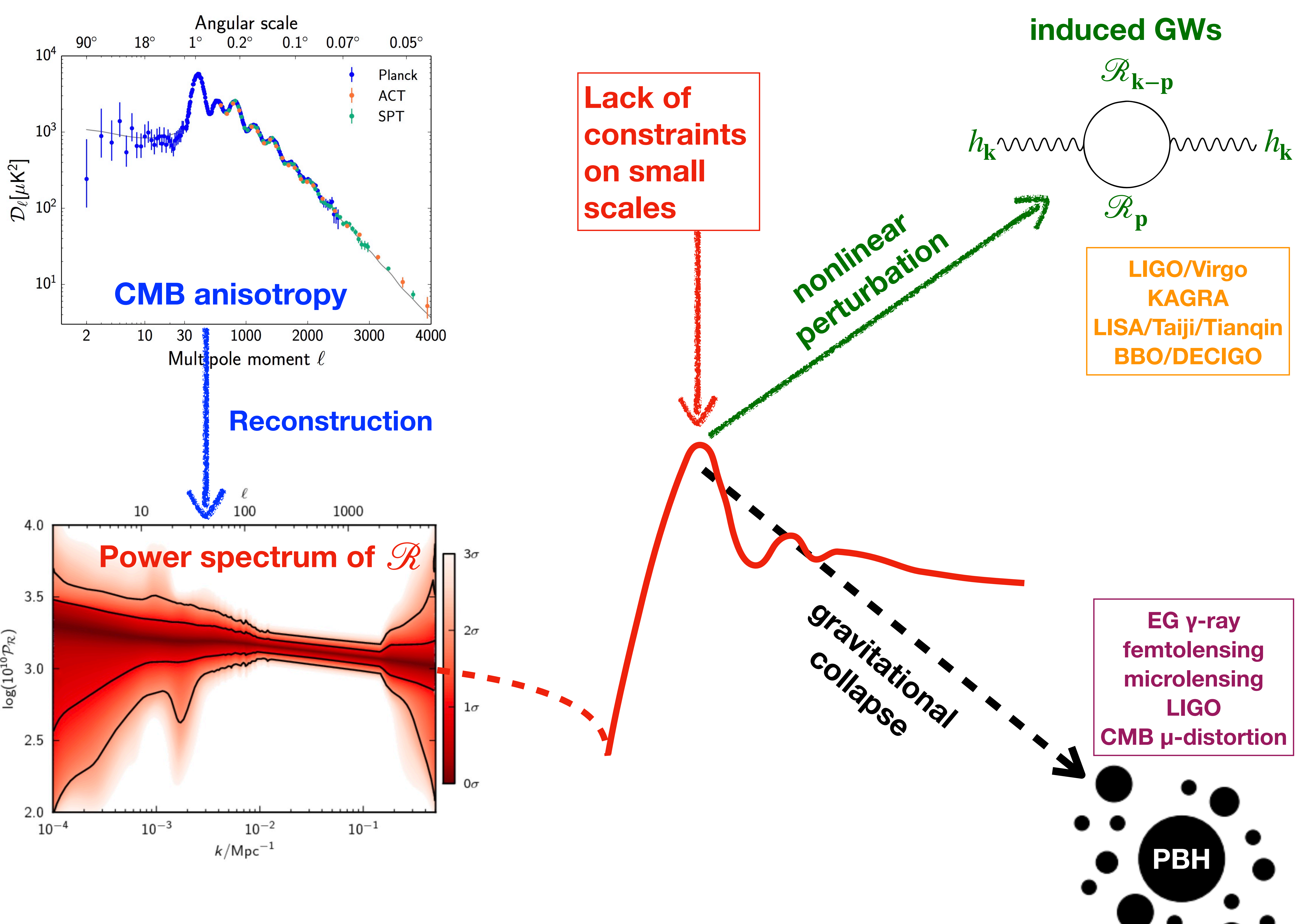


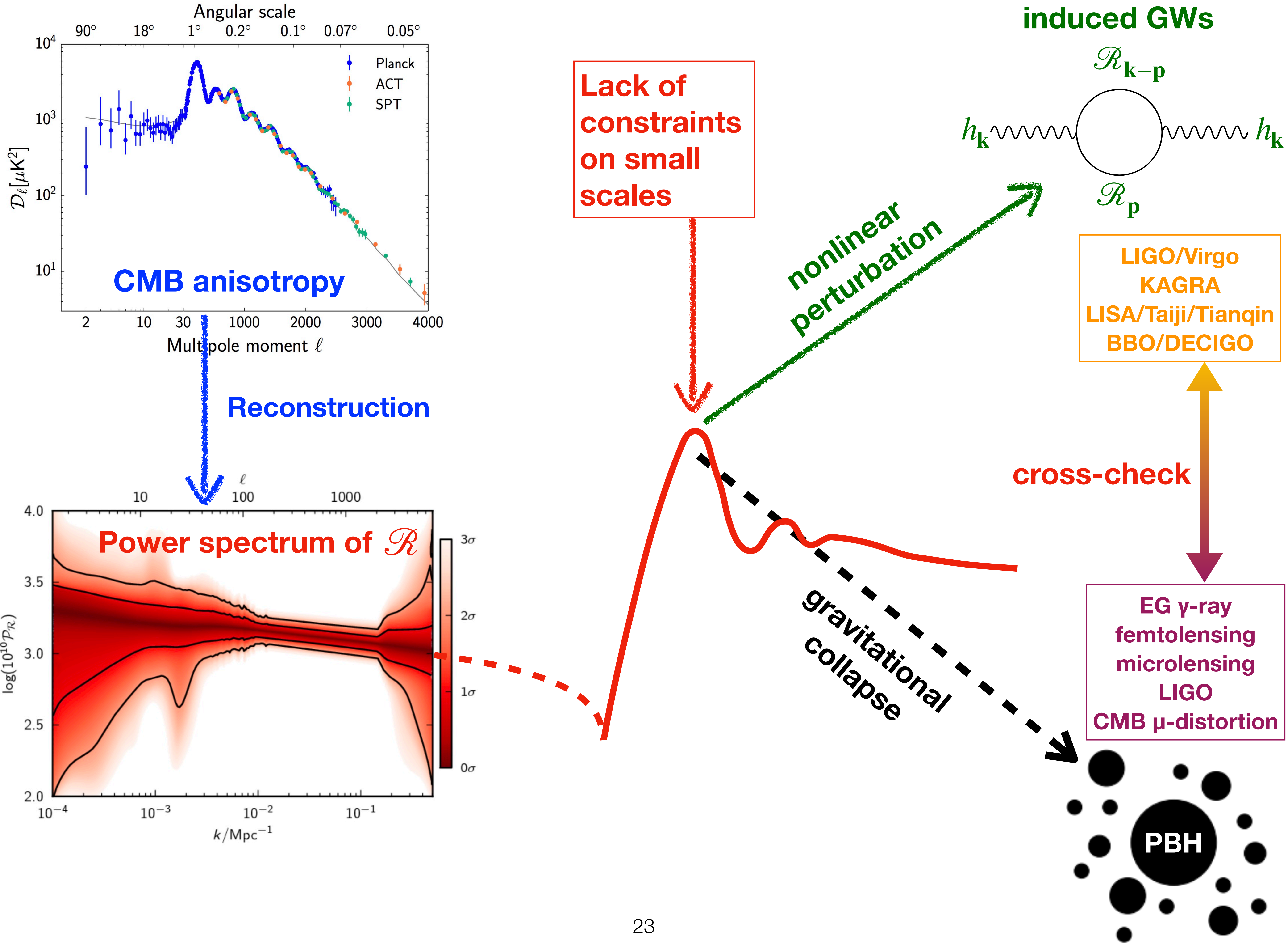
[Planck 2015, 4 knots]

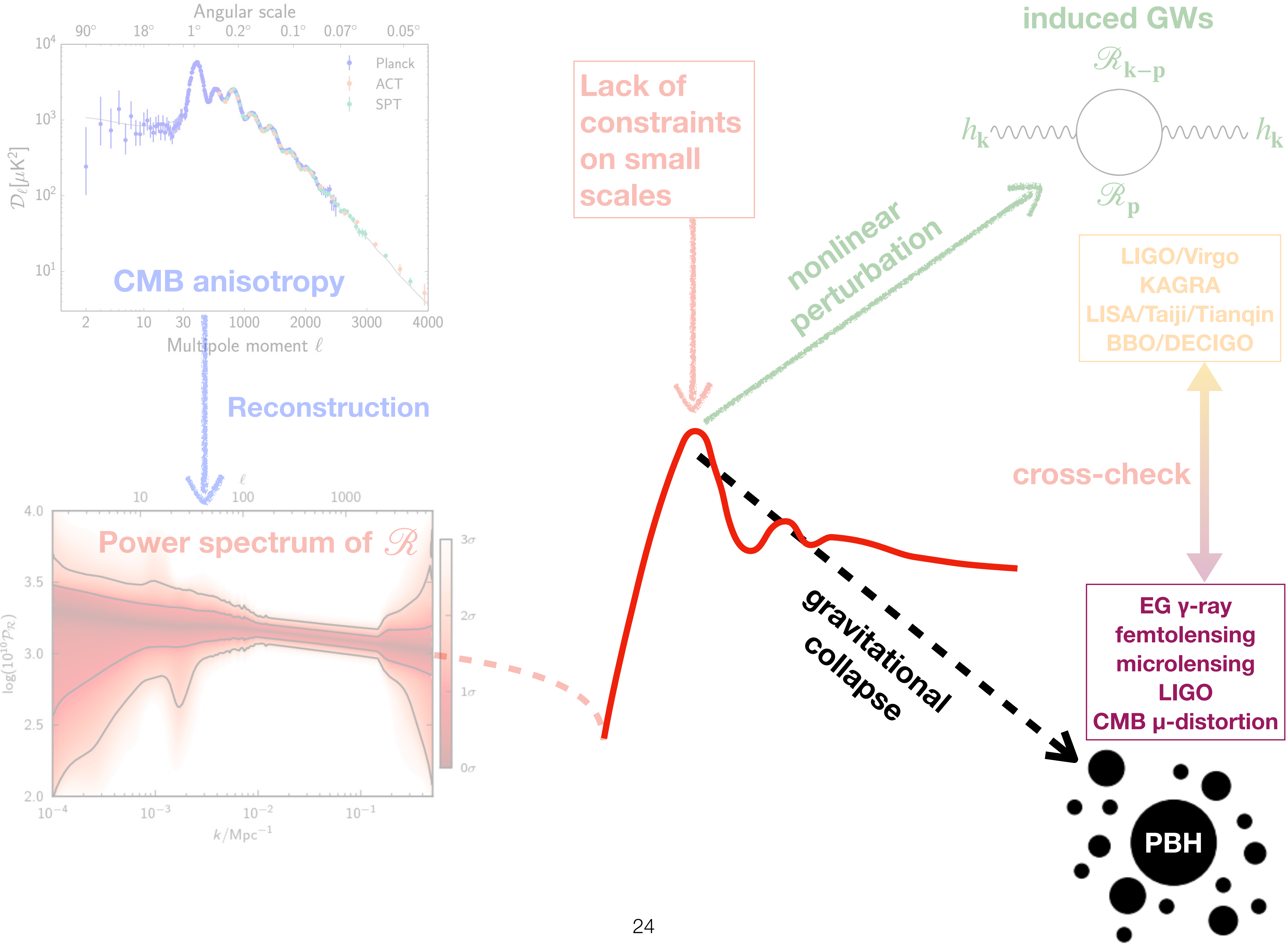


Angular scale
90° 18° 1° 0.2° 0.1° 0.07° 0.05°
Planck
ACT
SPT
$\mathcal{D}_\ell [\mu K^2]$
CMB anisotropy
2 10 30 1000 2000 3000 4000
Multipole moment ℓ
Reconstruction
Lack of constraints on small scales
ℓ
10 100 1000
Power spectrum of $\mathscr{R}$
$\log(10^{10}\mathcal{P}_\mathcal{R})$
3σ 2σ 1σ 0σ
k/Mpc^{-1}

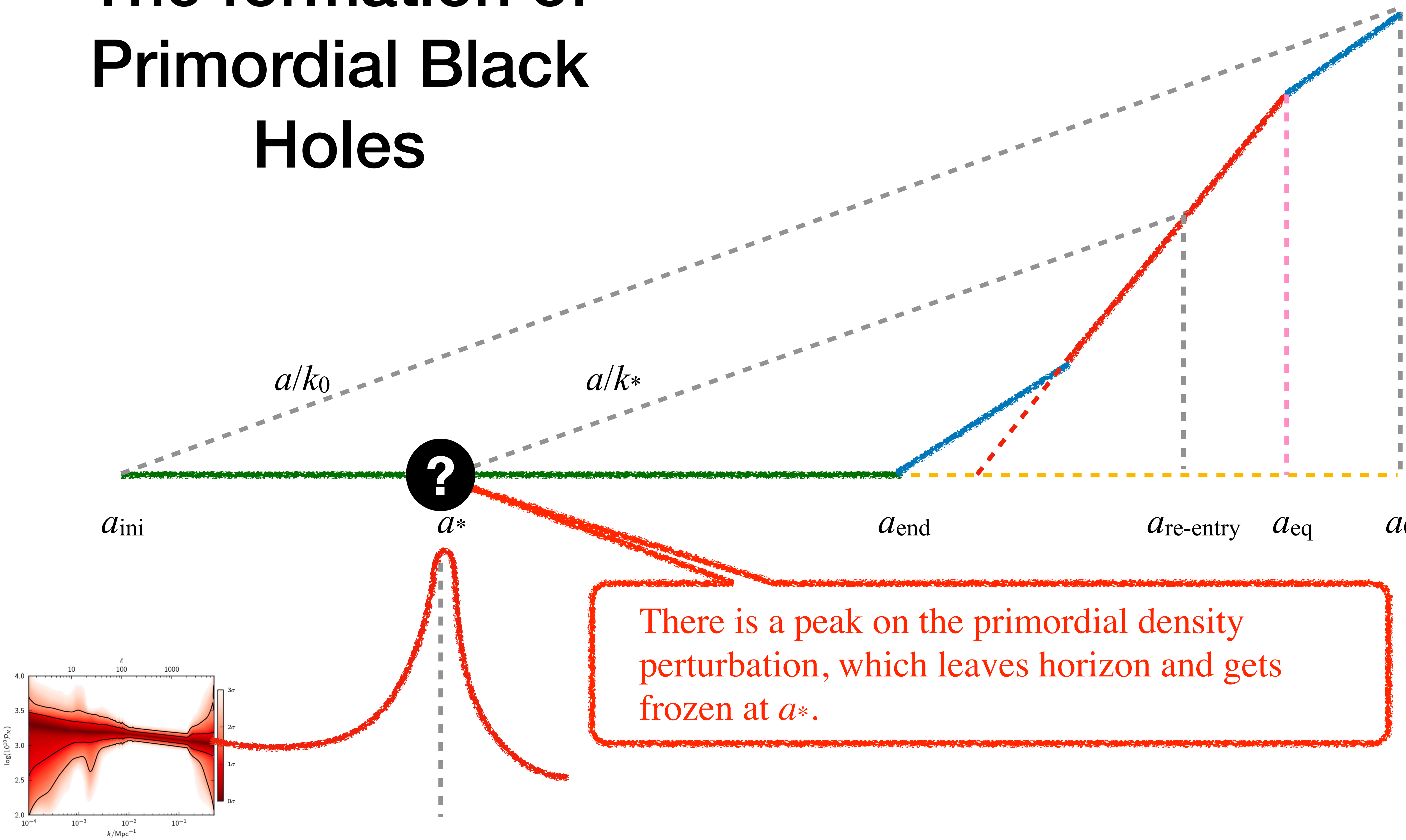

Angular scale
90° 18° 1° 0.2° 0.1° 0.07° 0.05°
Planck
ACT
SPT
$\mathcal{D}_\ell[\mu K^2]$
2 10 30 1000 2000 3000 4000
Multipole moment ℓ
CMB anisotropy
Reconstruction
ℓ
10 100 1000
Power spectrum of $\mathscr{R}$
$\log(10^{10}\mathcal{P}_\mathcal{R})$
k/Mpc^{-1}
3σ
2σ
1σ
0σ
Lack of constraints on small scales


Angular scale
90°
18°
1°
0.2°
0.1°
0.07°
0.05°
Planck
ACT
SPT
$\mathcal{D}_\ell[\mu K^2]$
CMB anisotropy
Multipole moment ℓ
Reconstruction
ℓ
Power spectrum of $\mathscr{R}$
$\log(10^{10}\mathcal{P}_{\mathscr{R}})$
k/Mpc^{-1}
3σ
2σ
1σ
0σ
Lack of constraints on small scales
nonlinear perturbation
induced GWs
$\mathscr{R}_{\mathbf{k-p}}$
$h_{\mathbf{k}}$
$h_{\mathbf{k}}$
$\mathscr{R}_{\mathbf{p}}$
LIGO/Virgo
KAGRA
LISA/Taiji/Tianqin
BBO/DECIGO
gravitational collapse
EG γ-ray
femtolensing
microlensing
LIGO
CMB μ-distortion
PBH


Angular scale
90° 18° 1° 0.2° 0.1° 0.07° 0.05°
Planck
ACT
SPT
$\mathcal{D}_\ell[\mu K^2]$
10^4 10^3 10^2 10^1
2 10 30 1000 2000 3000 4000
Multipole moment ℓ
CMB anisotropy
Reconstruction
ℓ
10 100 1000
Power spectrum of $\mathscr{R}$
$\log(10^{10}\mathcal{P}_{\mathcal{R}})$
4.0 3.5 3.0 2.5 2.0
3σ 2σ 1σ 0σ
10^{-4} 10^{-3} 10^{-2} 10^{-1}
k/Mpc^{-1}
Lack of constraints on small scales
nonlinear perturbation
gravitational collapse
induced GWs
$\mathscr{R}_{\mathbf{k-p}}$
$h_{\mathbf{k}}$
$h_{\mathbf{k}}$
$\mathscr{R}_{\mathbf{p}}$
LIGO/Virgo
KAGRA
LISA/Taiji/Tianqin
BBO/DECIGO
cross-check
EG γ-ray
femtolensing
microlensing
LIGO
CMB μ-distortion
PBH


Angular scale
90° 18° 1° 0.2° 0.1° 0.07° 0.05°
Planck
ACT
SPT
CMB anisotropy
2 10 30 1000 2000 3000 4000
Multipole moment ℓ
Reconstruction
Power spectrum of ℛ
k/Mpc⁻¹
Lack of constraints on small scales
nonlinear perturbation
induced GWs
ℛk−p
ℛp
hk
hk
LIGO/Virgo
KAGRA
LISA/Taiji/Tianqin
BBO/DECIGO
cross-check
gravitational collapse
EG γ-ray
femtolensing
microlensing
LIGO
CMB μ-distortion
PBH

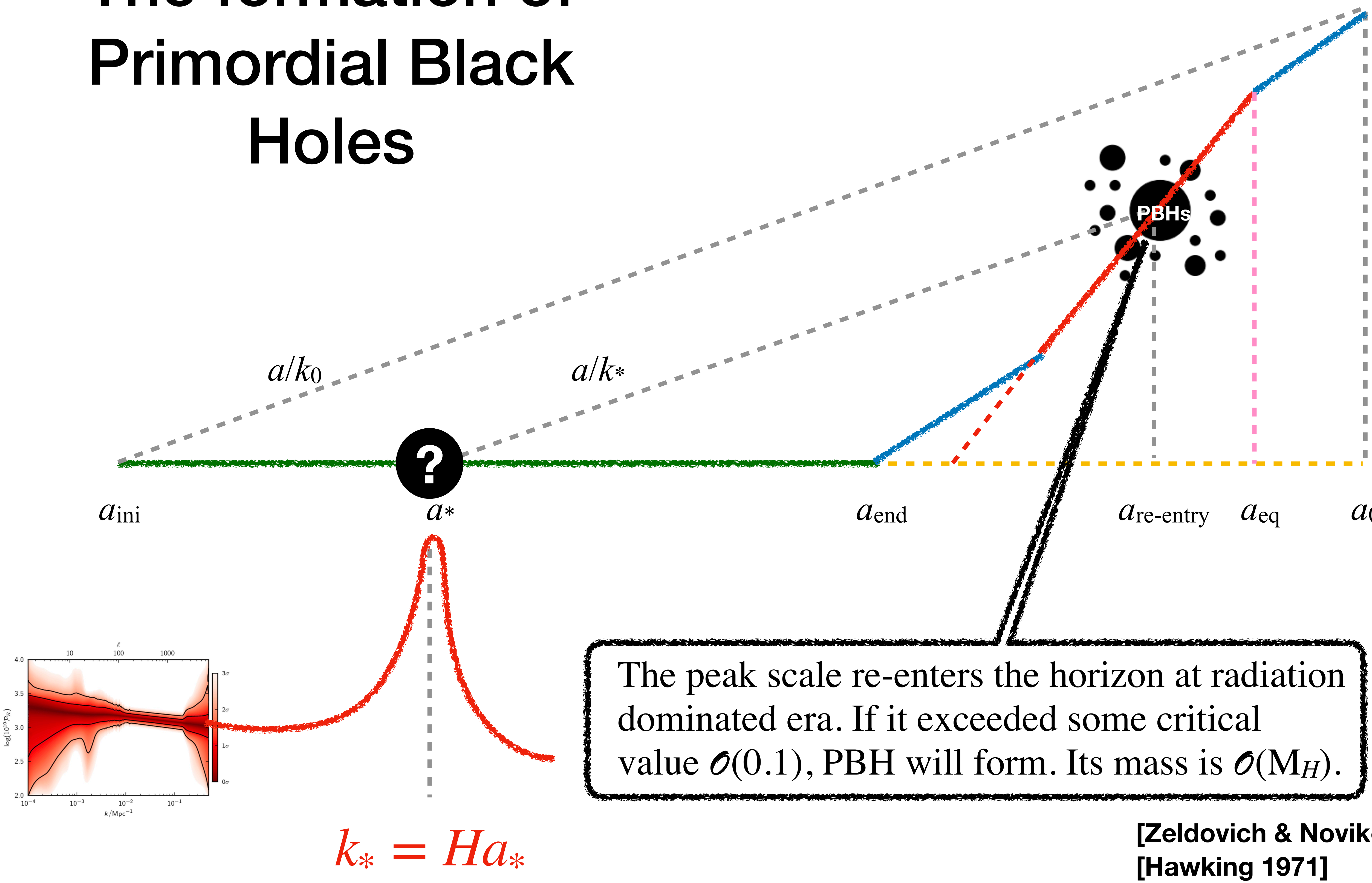
The formation of Primordial Black Holes



There is a peak on the primordial density perturbation, which leaves horizon and gets frozen at a_*.

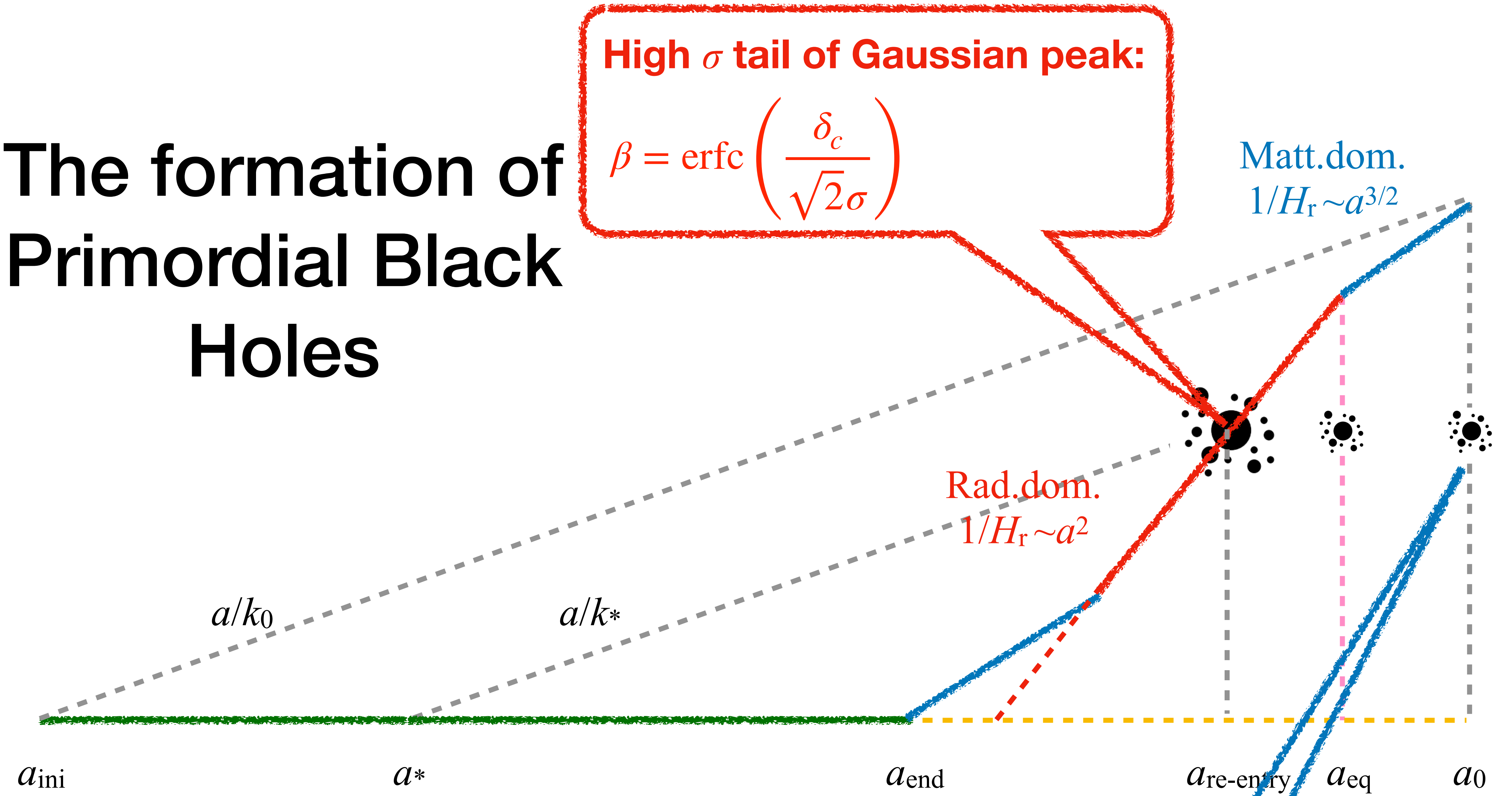
$$k_* = Ha_*$$

The formation of Primordial Black Holes



[Zeldovich & Novikov 1966]
[Hawking 1971]
[Carr & Hawking 1974]

The formation of Primordial Black Holes



$$\Omega_{PBH} = \beta \frac{a_{eq}}{a_{re}} = \beta \frac{a_{eq}}{a_0} \frac{a_0}{a_{re}} \simeq \beta \Omega_r (1 + z_{re}(M))$$

$$M = \frac{c^3}{G H_{re}} = \frac{c^3}{G \Omega_r^{1/2} (1+z)^2 H_0}$$

Redshifted after equality:

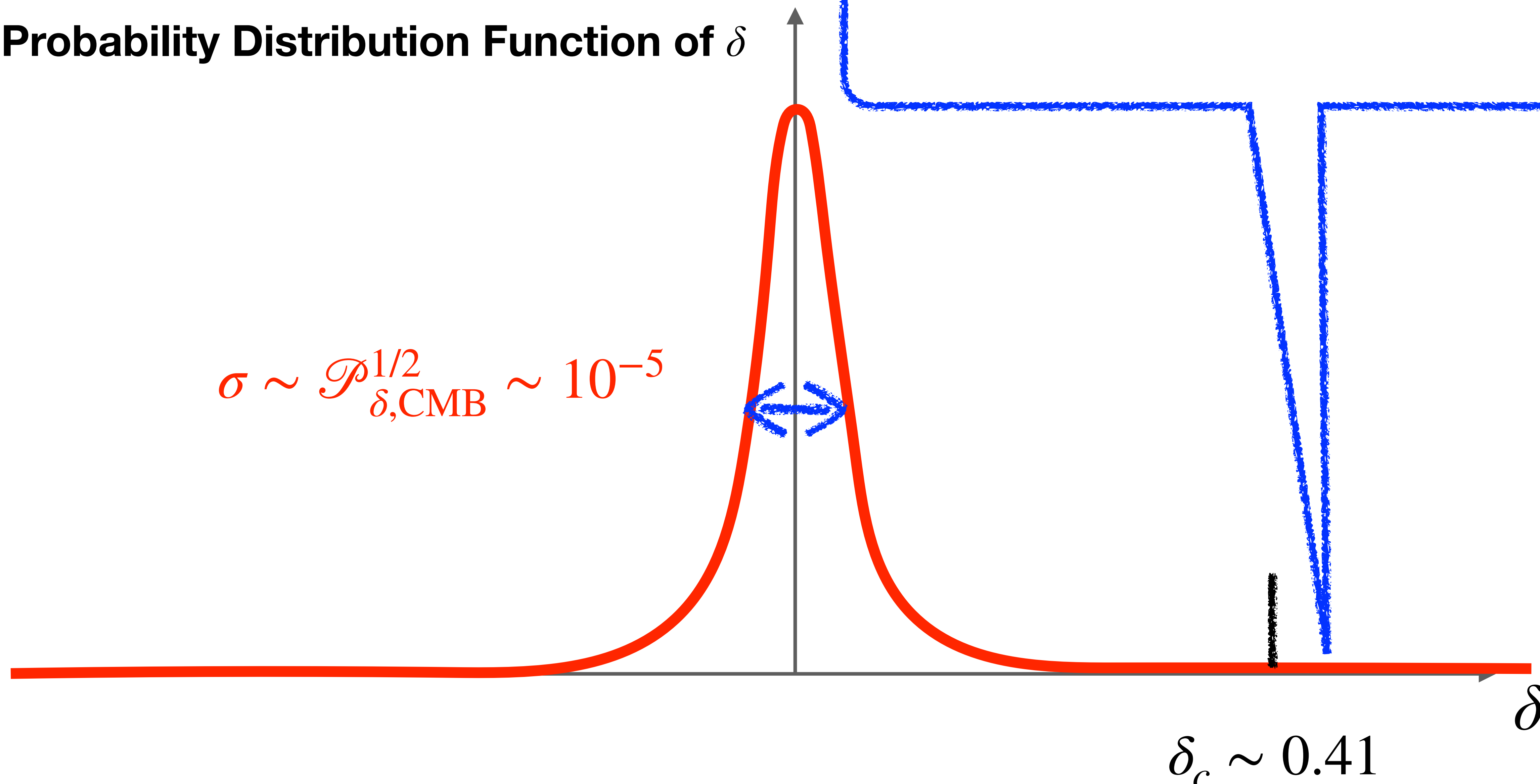
$$f_{PBH} \equiv \frac{\Omega_{PBH}}{\Omega_{CDM}} = 4.11 \times 10^8 \beta(M) \left(\frac{M}{M_\odot}\right)^{-1/2}$$

CMB-scale perturbations are negligible

PBH formation when $\delta > \delta_c$.

$$\beta = \mathrm{erfc}\frac{\delta_c}{\sqrt{2}\sigma} \simeq \exp\left(-\frac{\delta_c^2}{2\sigma^2}\right) \sim \exp\left(-10^{10}\right).$$

Negligible!



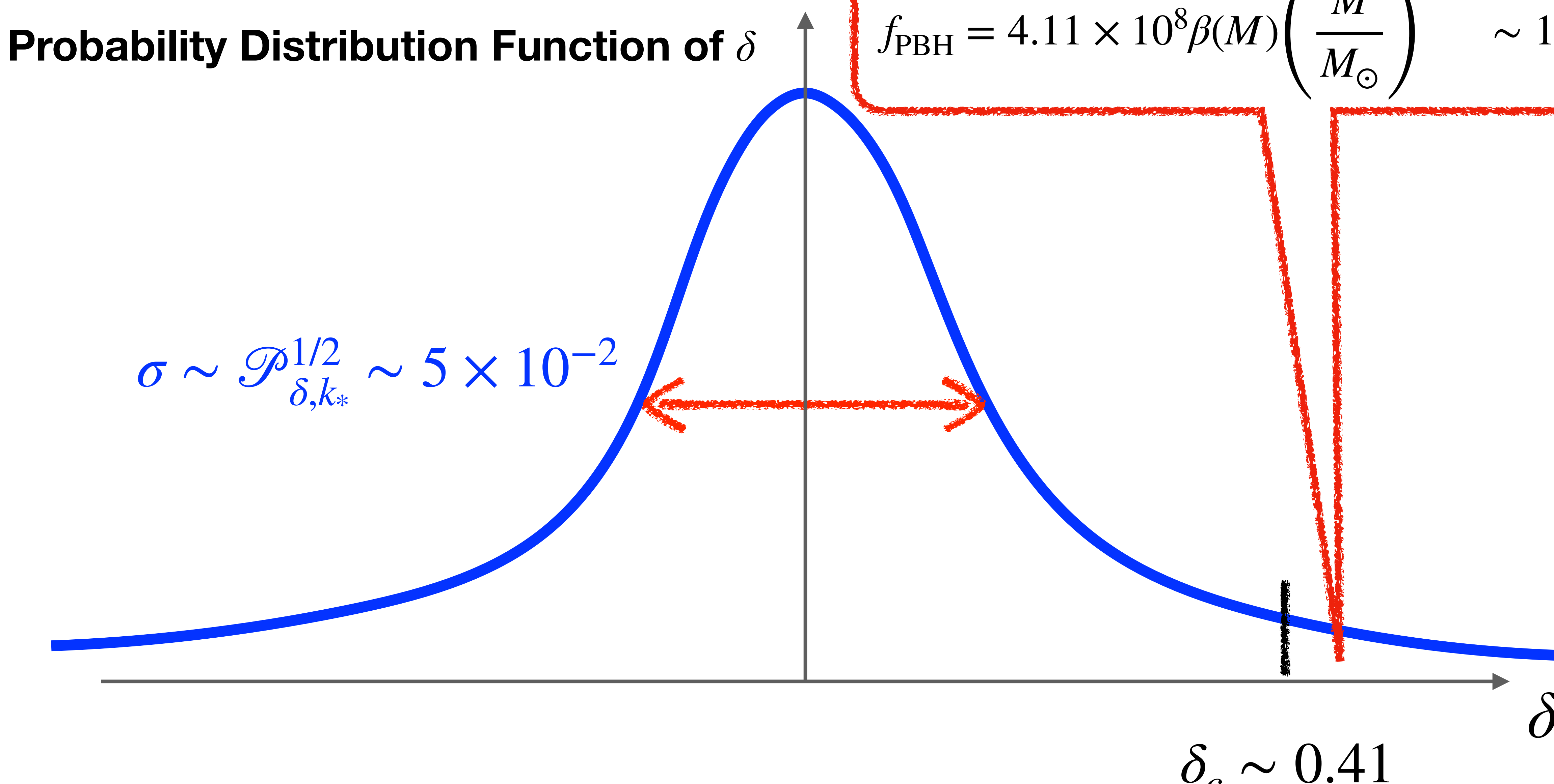
PBH=DM requires large perturbations (on small scales)

PBH formation when $\delta > \delta_c$.

$$\beta = \mathrm{erfc}\frac{\delta_c}{\sqrt{2}\sigma} \simeq \exp\left(-\frac{\delta_c^2}{2\sigma^2}\right) \sim 2.5 \times 10^{-15}$$

PBH=DM (For $M \sim 10^{21}$ g **)**

$$f_{\mathrm{PBH}} = 4.11 \times 10^8 \beta(M) \left(\frac{M}{M_\odot}\right)^{-1/2} \sim 1$$



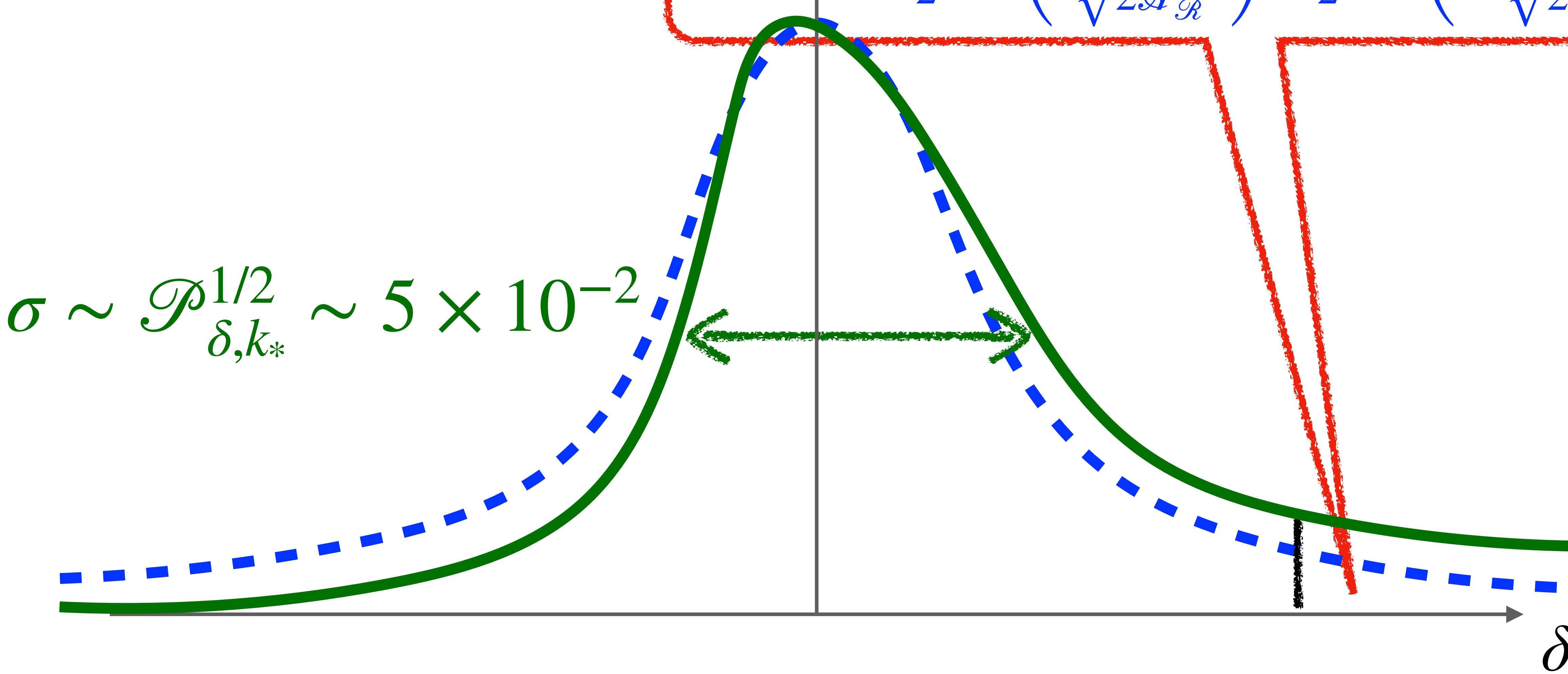
NG to enhance PBH

PBH formation when $\delta > \delta_c$.

(Positive) Non-Gaussianity can increase the PBH production. (If we fix the variance)

$$\mathcal{R}_{g\pm}(\mathcal{R}) = \frac{1}{2} f_{\rm NL}^{-1} \left(-1 \pm \sqrt{1 + 4 f_{\rm NL} \left(f_{\rm NL} \mathcal{A}_{\mathcal{R}} + \mathcal{R} \right)} \right) .$$

$$\beta = \frac{1}{2} \mathrm{erfc} \left(\frac{\mathcal{R}_{g+}(\mathcal{R}_c)}{\sqrt{2\mathcal{A}_{\mathcal{R}}}} \right) + \frac{1}{2} \mathrm{erfc} \left(- \frac{\mathcal{R}_{g-}(\mathcal{R}_c)}{\sqrt{2\mathcal{A}_{\mathcal{R}}}} \right) .$$



[Young & Byrnes, JCAP, 1307.4995]

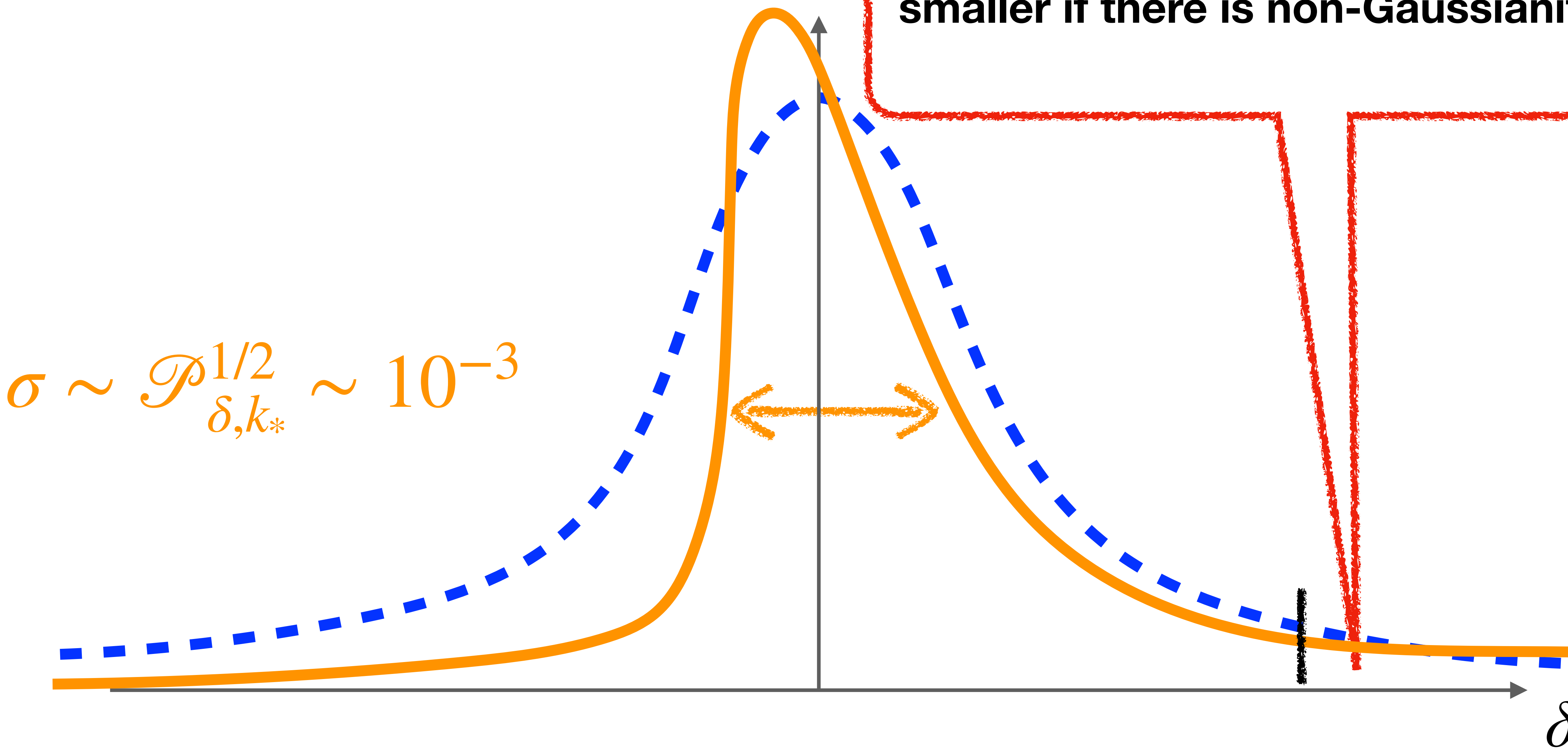
$\delta_c \sim 0.41$

PBH=DM w/. NG

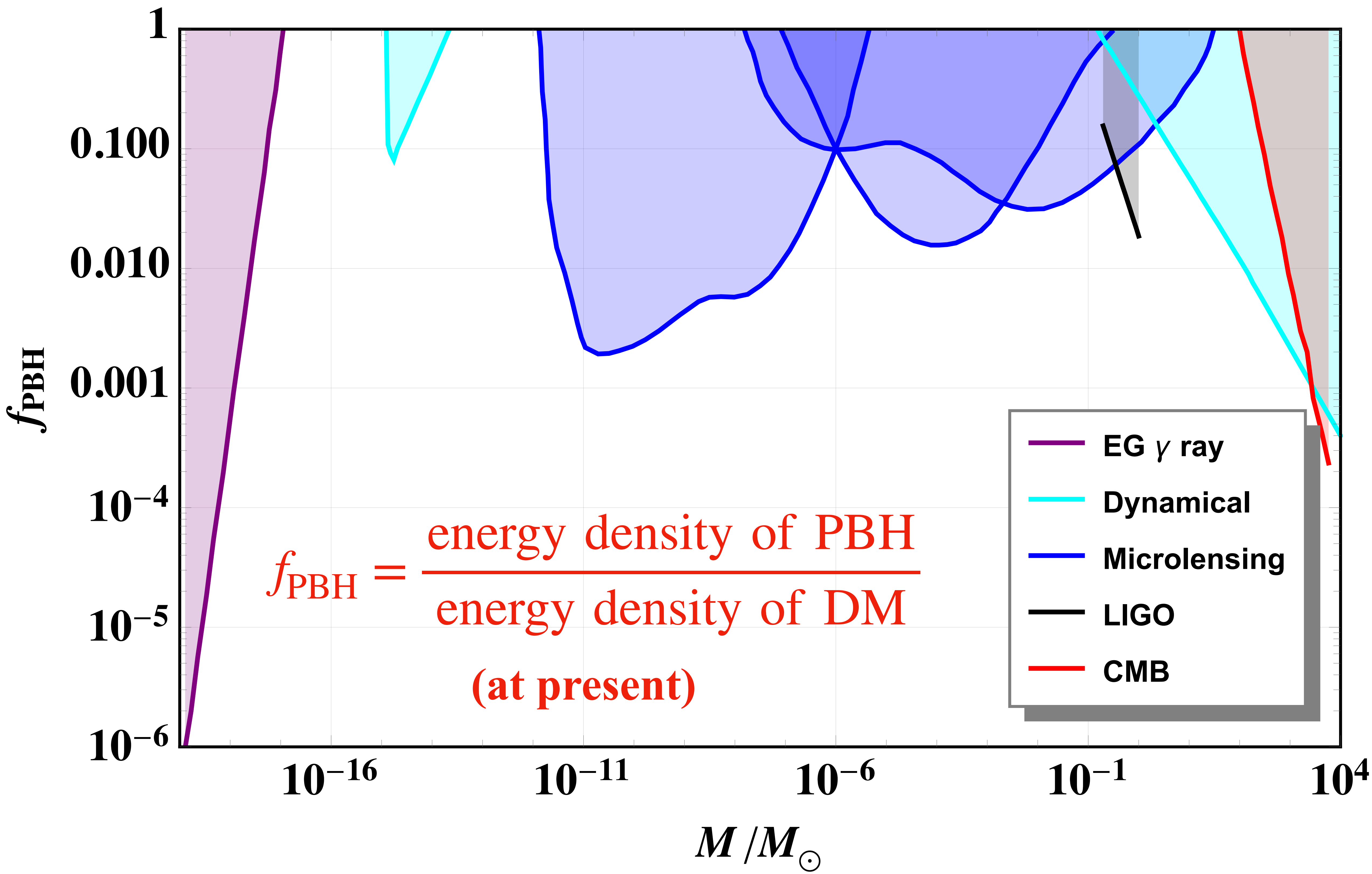
PBH formation when $\delta > \delta_c$.

$$f_{\mathrm{PBH}} = 4.11 \times 10^8 \beta(M) \left(\frac{M}{M_\odot} \right)^{-1/2} \sim 1$$

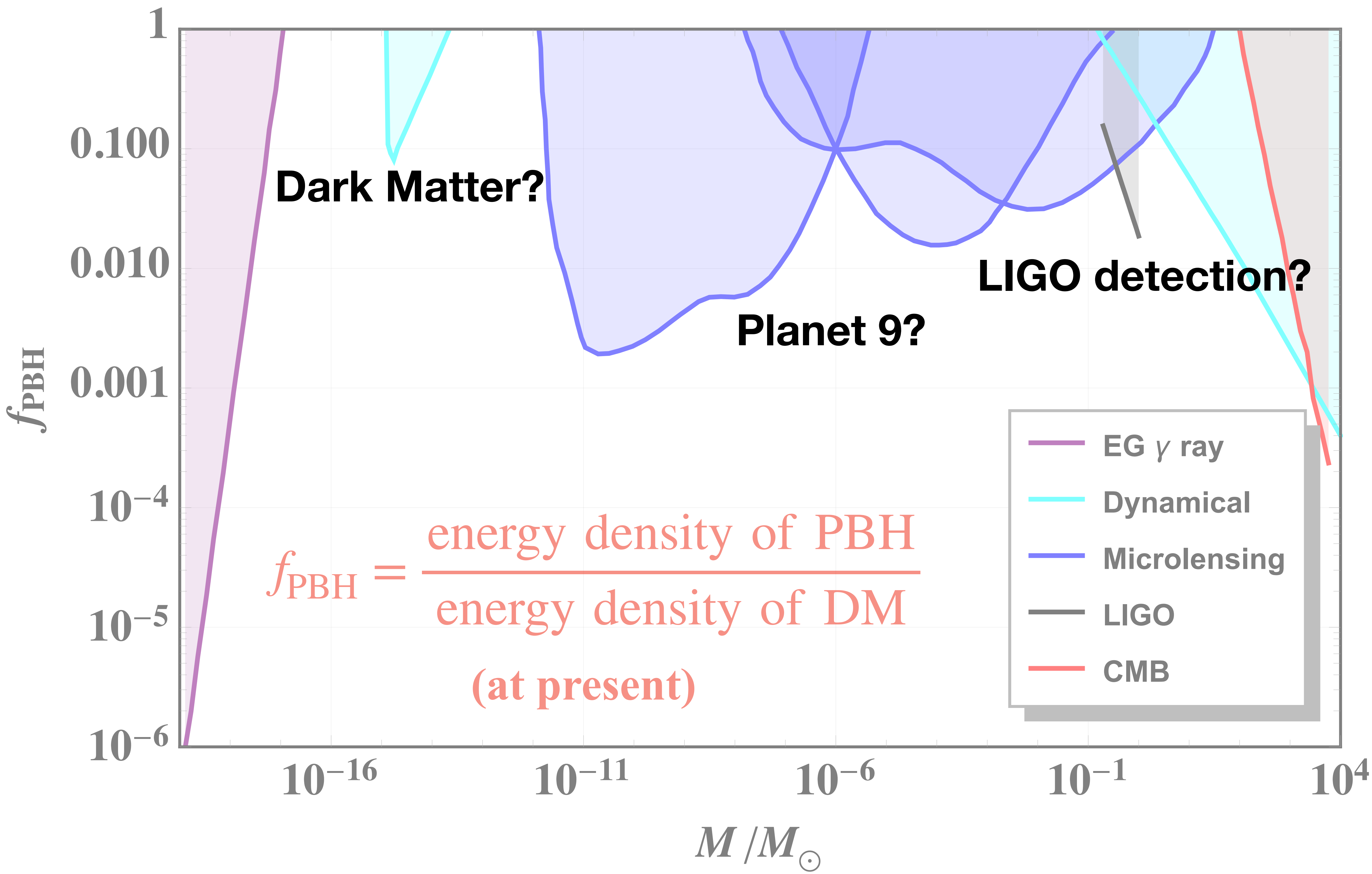
If we fix the PBH abundance (to be all dark matter), the variance must be smaller if there is non-Gaussianity.



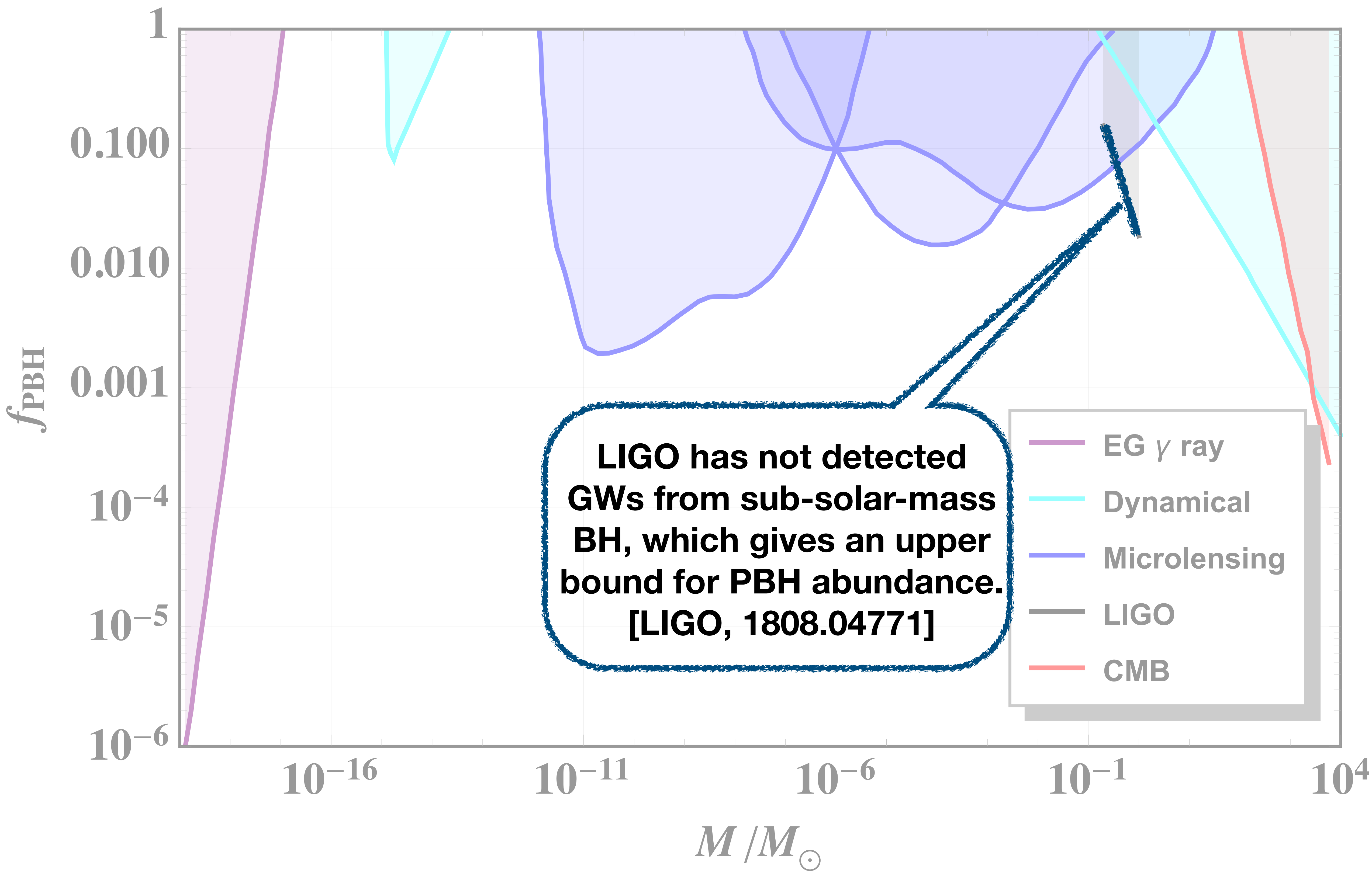
[Cai, SP & Sasaki, PRL122, 201101]

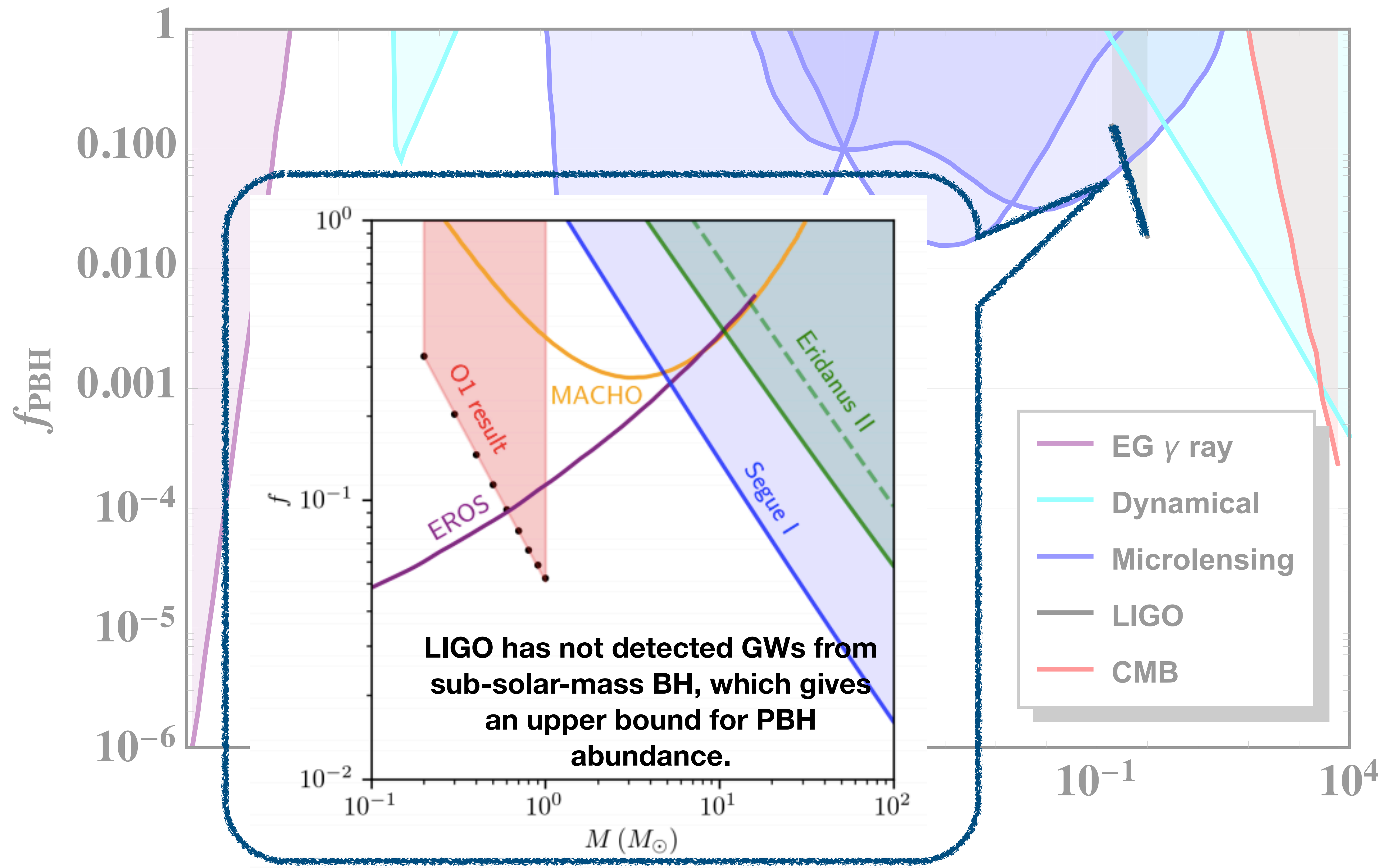


$f_{\rm PBH}$
1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_\odot$
EG γ ray
Dynamical
Microlensing
LIGO
CMB
$f_{\rm PBH} = \dfrac{\text{energy density of PBH}}{\text{energy density of DM}}$
(at present)

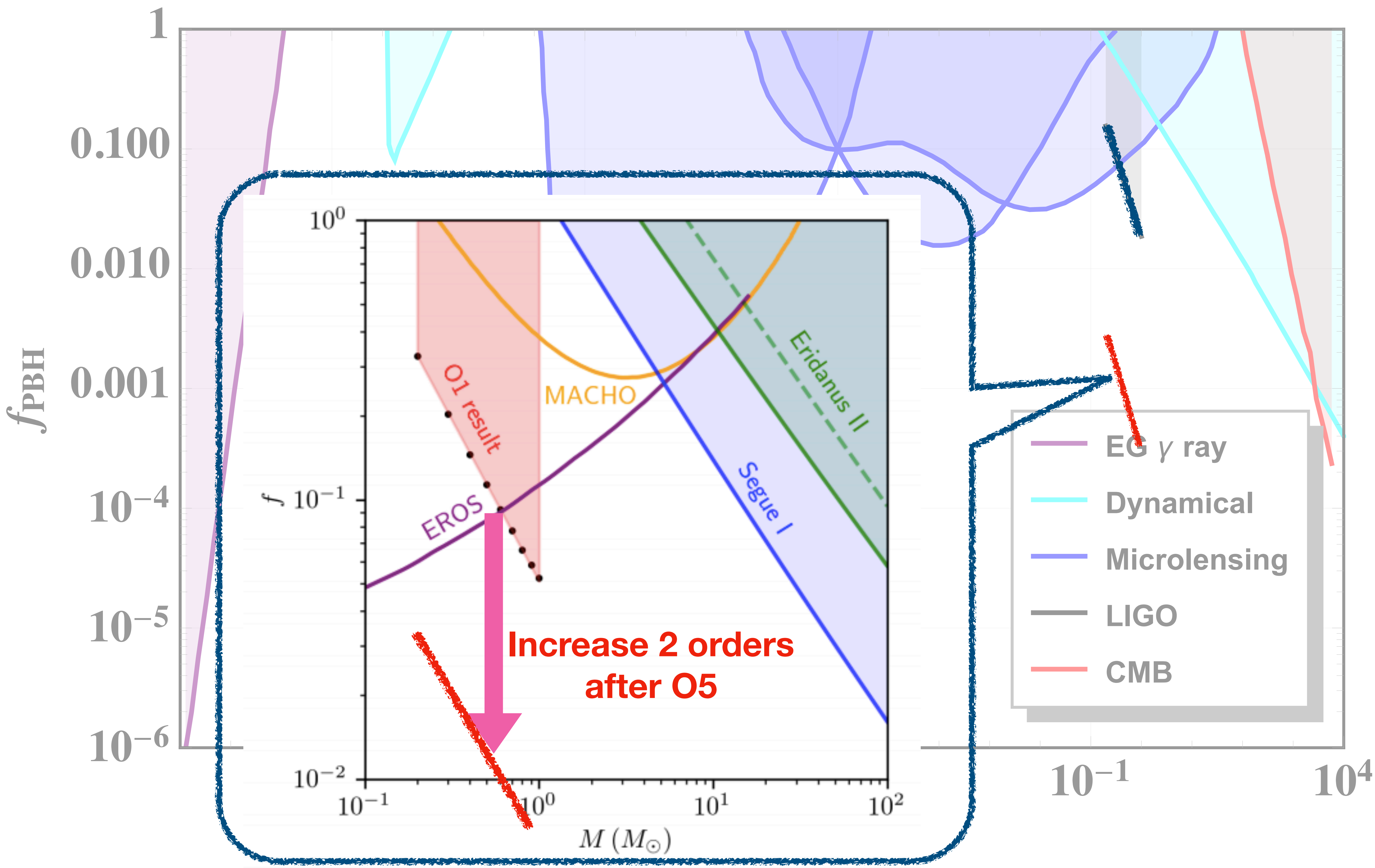


1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
f_{PBH}
Dark Matter?
Planet 9?
LIGO detection?
EG γ ray
Dynamical
Microlensing
LIGO
CMB
$f_{\mathrm{PBH}} = \frac{\text{energy density of PBH}}{\text{energy density of DM}}$
(at present)
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_{\odot}$


LIGO has not detected GWs from sub-solar-mass BH, which gives an upper bound for PBH abundance. [LIGO, 1808.04771]
EG γ ray
Dynamical
Microlensing
LIGO
CMB
f_{PBH}
1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_{\odot}$

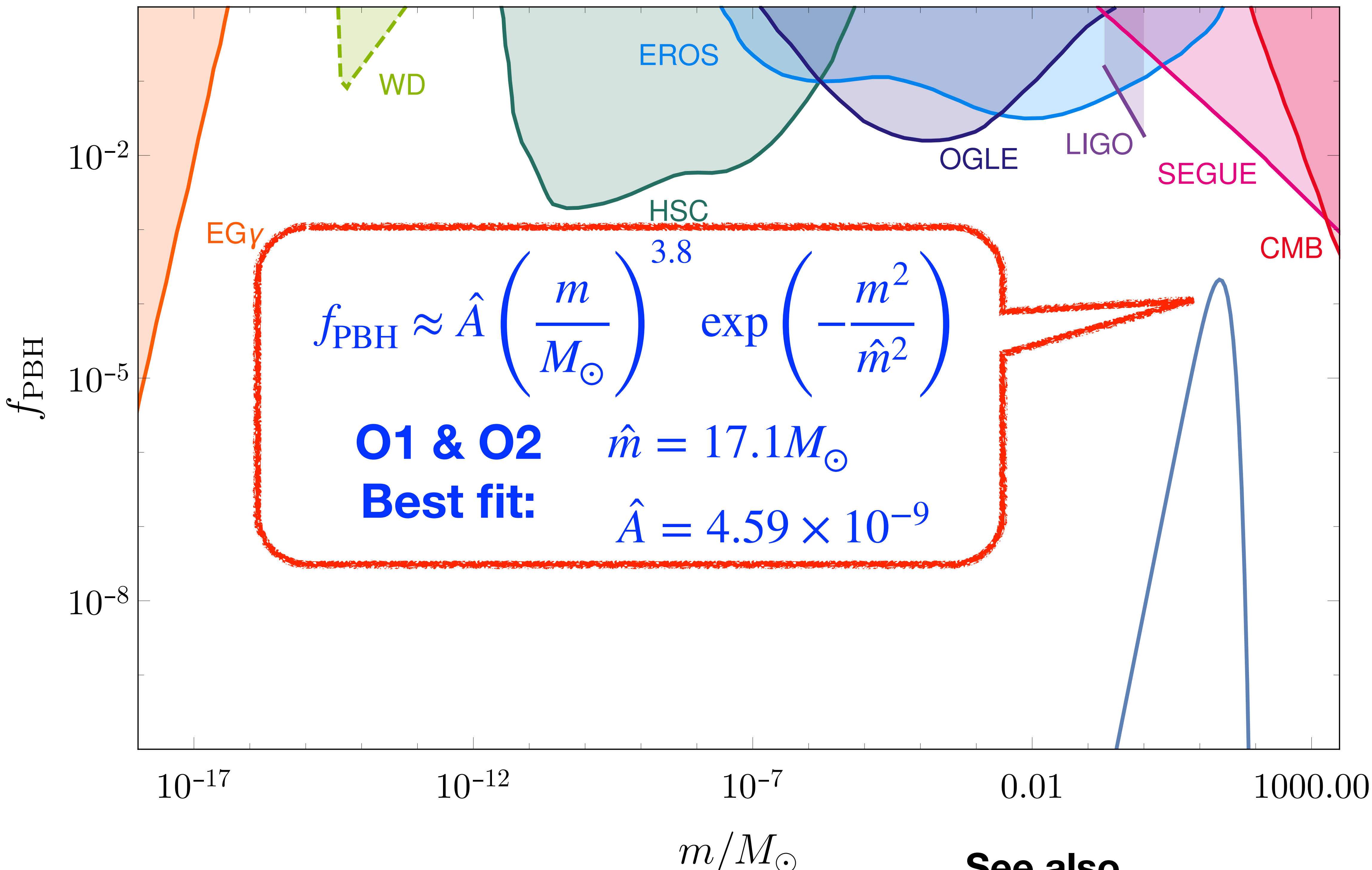


[LIGO,1808.04771]



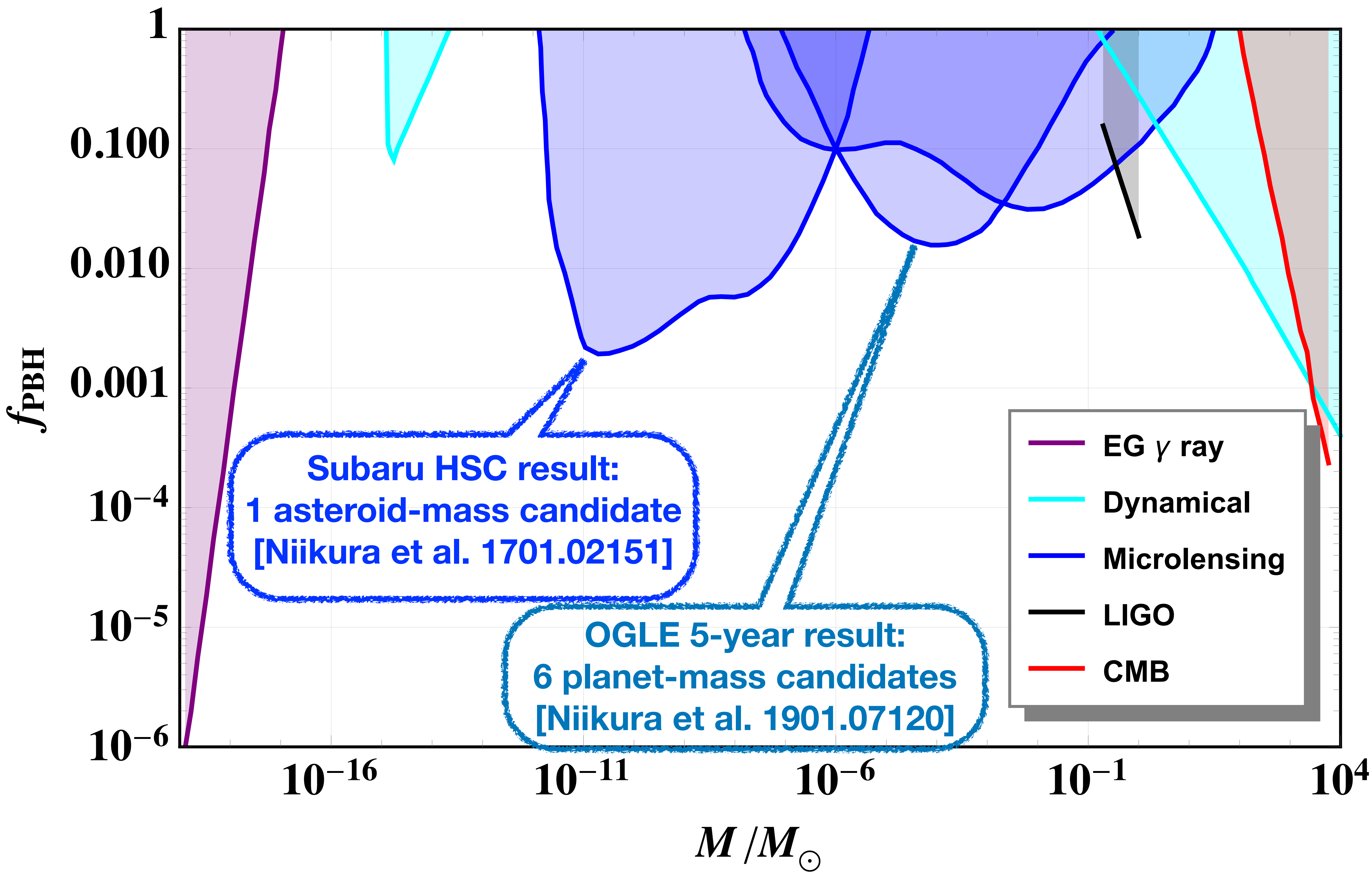
[LIGO,1808.04771]

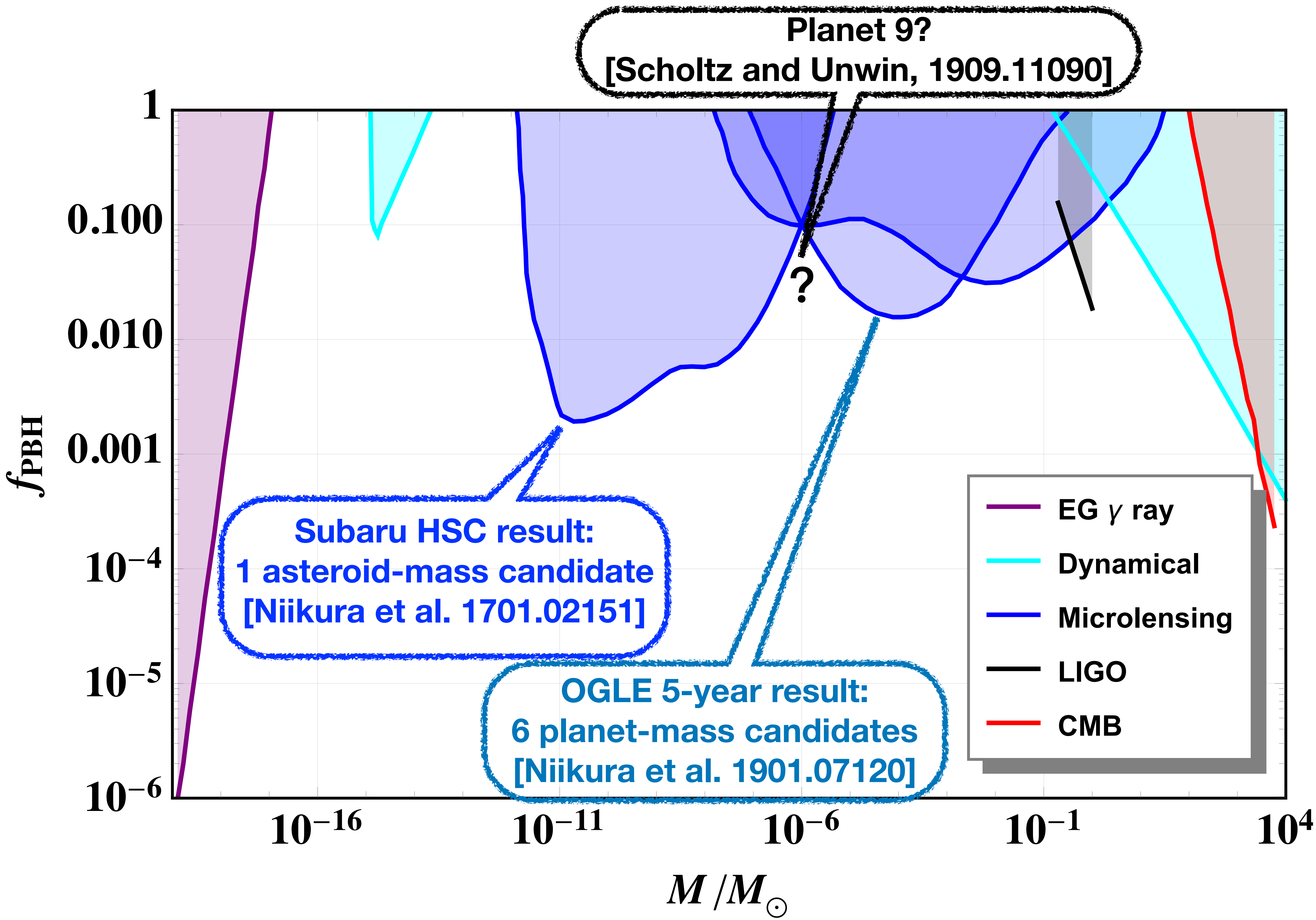
IGWs of LIGO-PBH

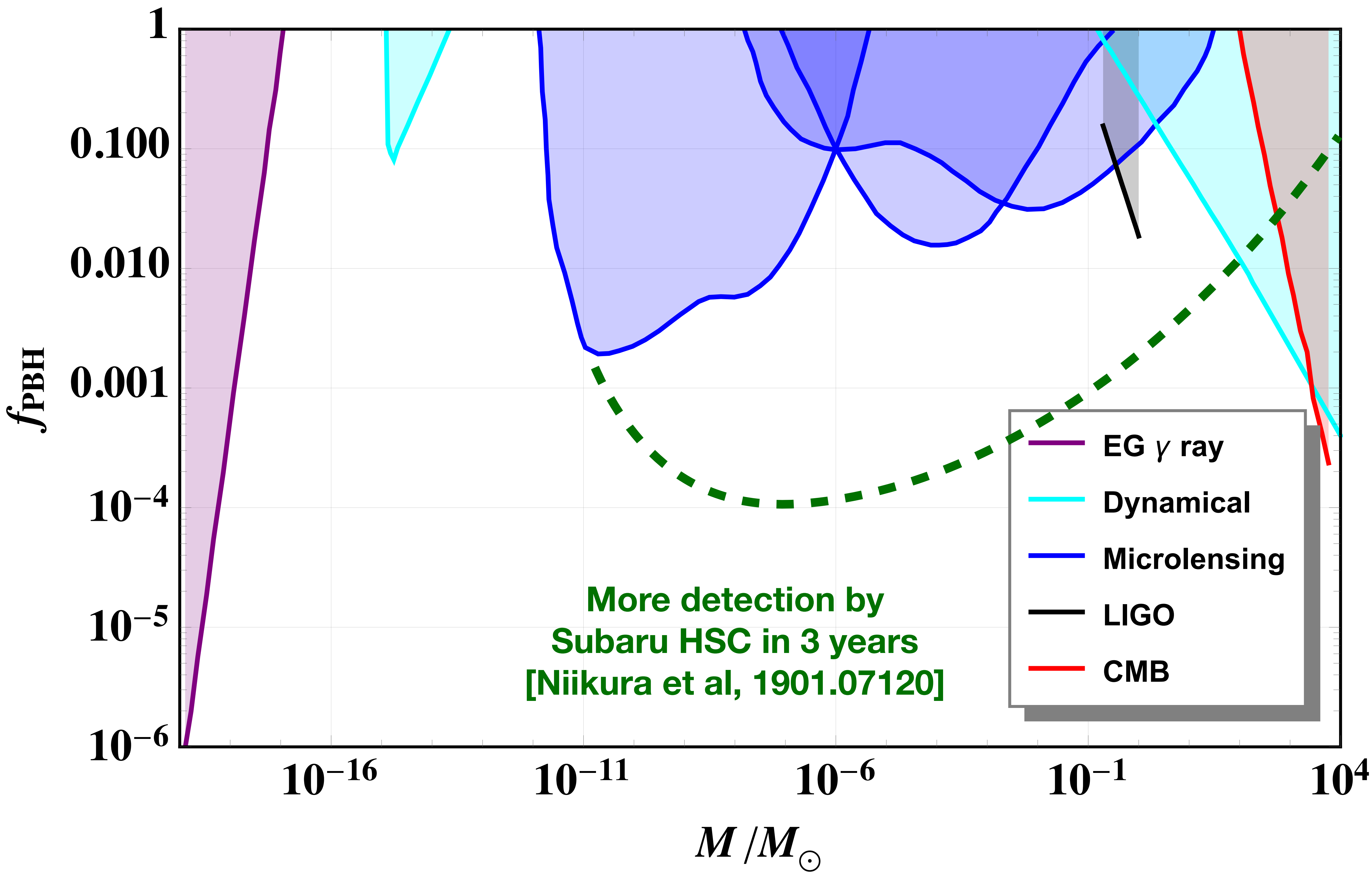


[Cai, SP, Wang and Yang, JCAP1910, 059]

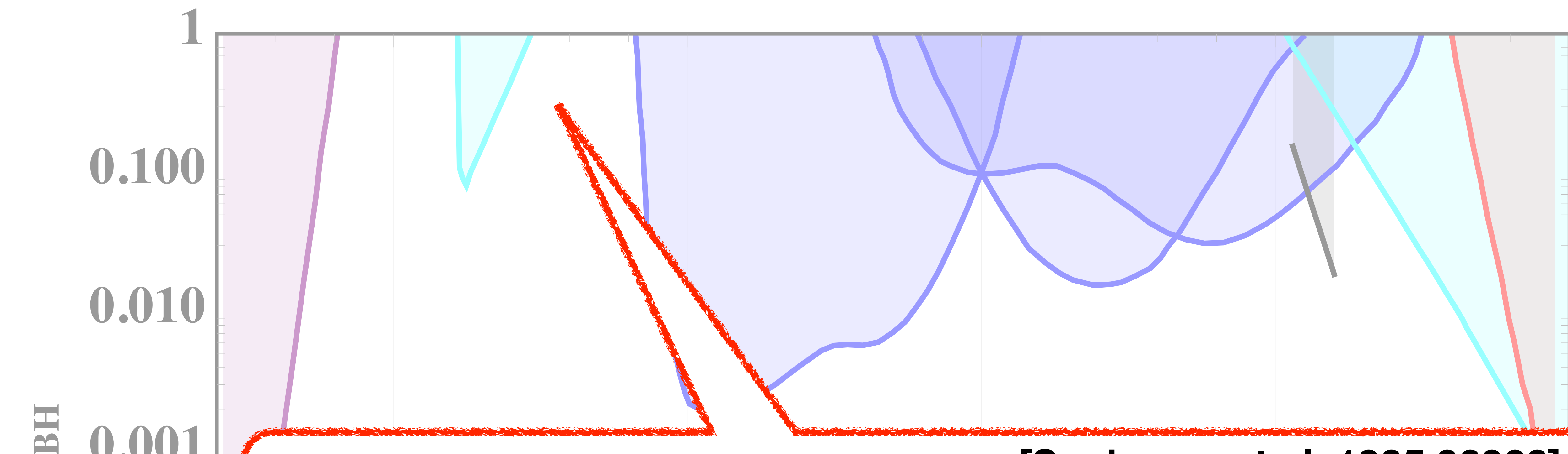
See also
[De Luca et al, 2005.05641]


1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
f_{PBH}
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_\odot$
Subaru HSC result:
1 asteroid-mass candidate
[Niikura et al. 1701.02151]
OGLE 5-year result:
6 planet-mass candidates
[Niikura et al. 1901.07120]
EG γ ray
Dynamical
Microlensing
LIGO
CMB


Planet 9?
[Scholtz and Unwin, 1909.11090]
?
1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
$f_{\rm PBH}$
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_\odot$
Subaru HSC result:
1 asteroid-mass candidate
[Niikura et al. 1701.02151]
OGLE 5-year result:
6 planet-mass candidates
[Niikura et al. 1901.07120]
EG γ ray
Dynamical
Microlensing
LIGO
CMB



1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
f_{PBH}
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_{\odot}$
More detection by
Subaru HSC in 3 years
[Niikura et al, 1901.07120]
EG γ ray
Dynamical
Microlensing
LIGO
CMB



No sensitivity on small scales

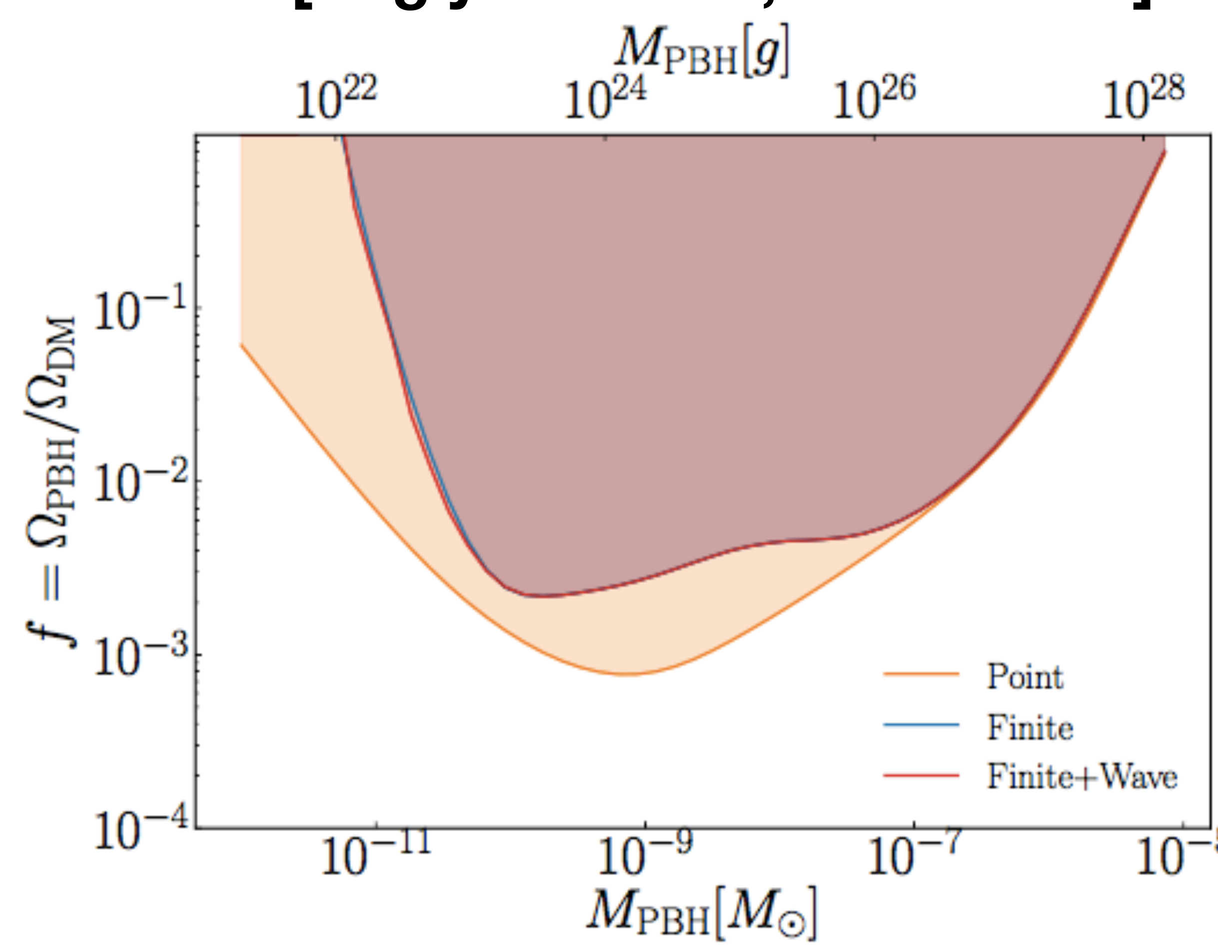
Finite Size Effect:

$$M < 5 \times 10^{-10} M_\odot \left(\frac{R_s}{R\odot}\right)^2$$

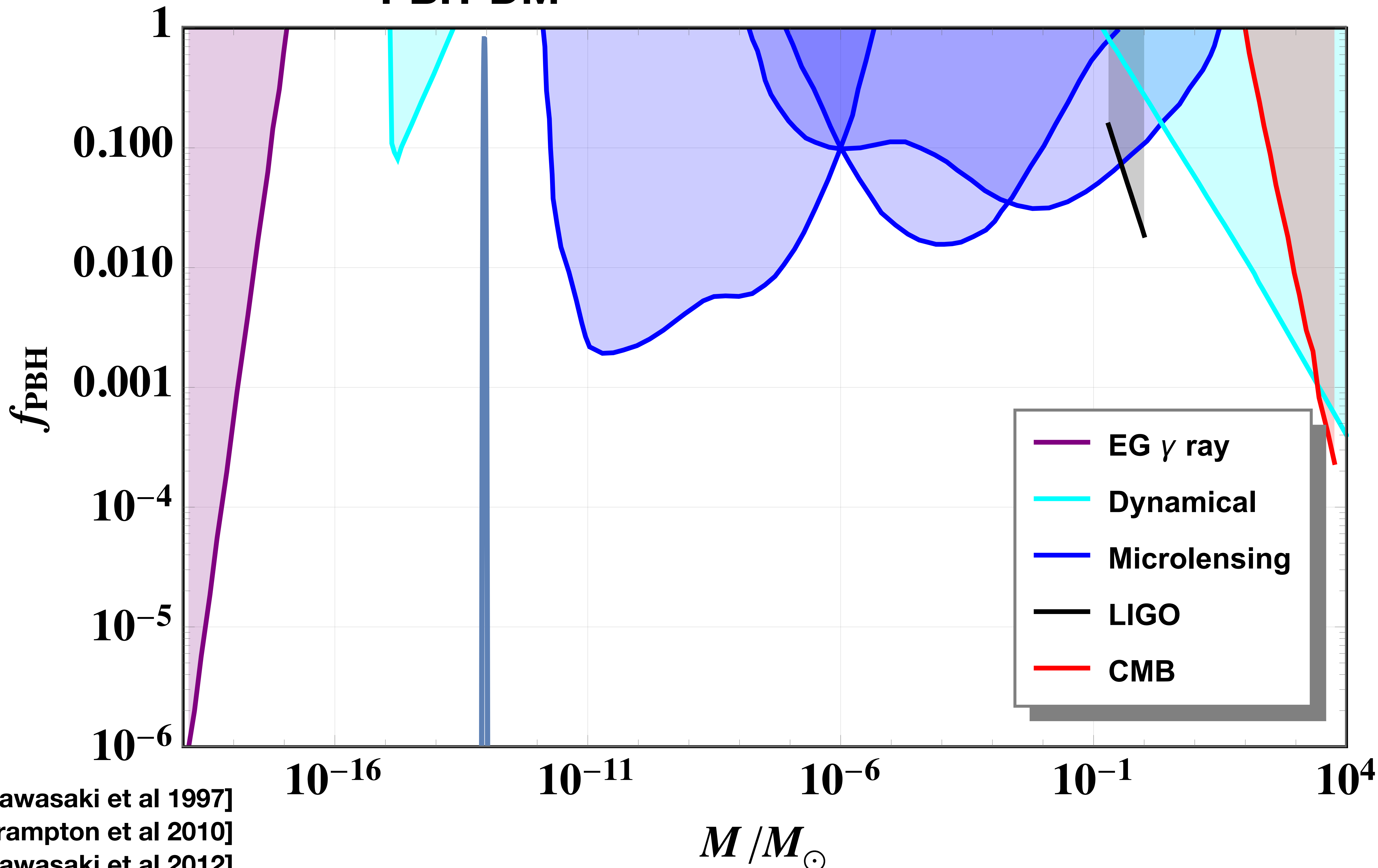
Wave effect:

$$M < 4 \times 10^{-10} M_\odot \left(\frac{\lambda_s}{\mu m}\right)$$

[Sugiyama et al, 1905.06066]



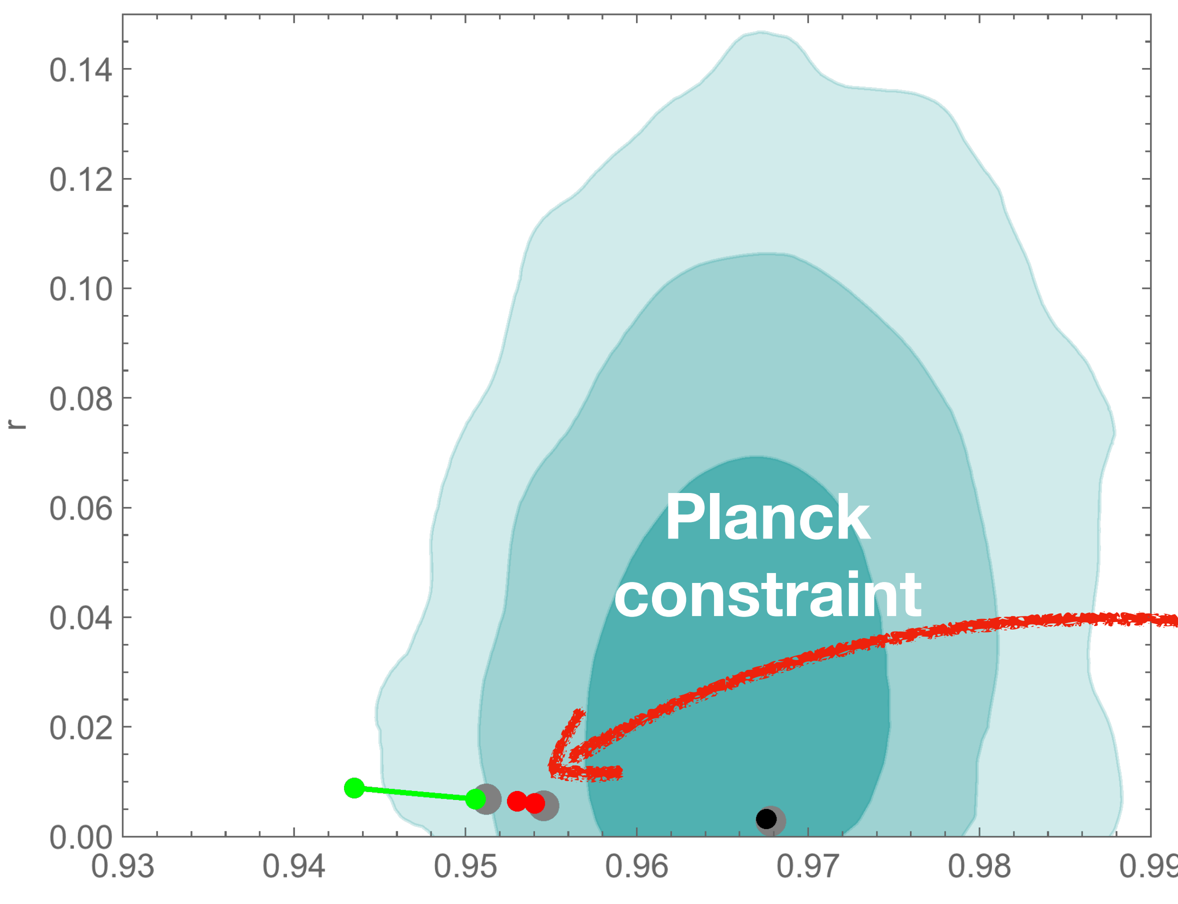
PBH-DM



[Kawasaki et al 1997]
[Frampton et al 2010]
[Kawasaki et al 2012]
[Inomata et al 2017]
[SP et al 2017]
[Cai et al 2018] …

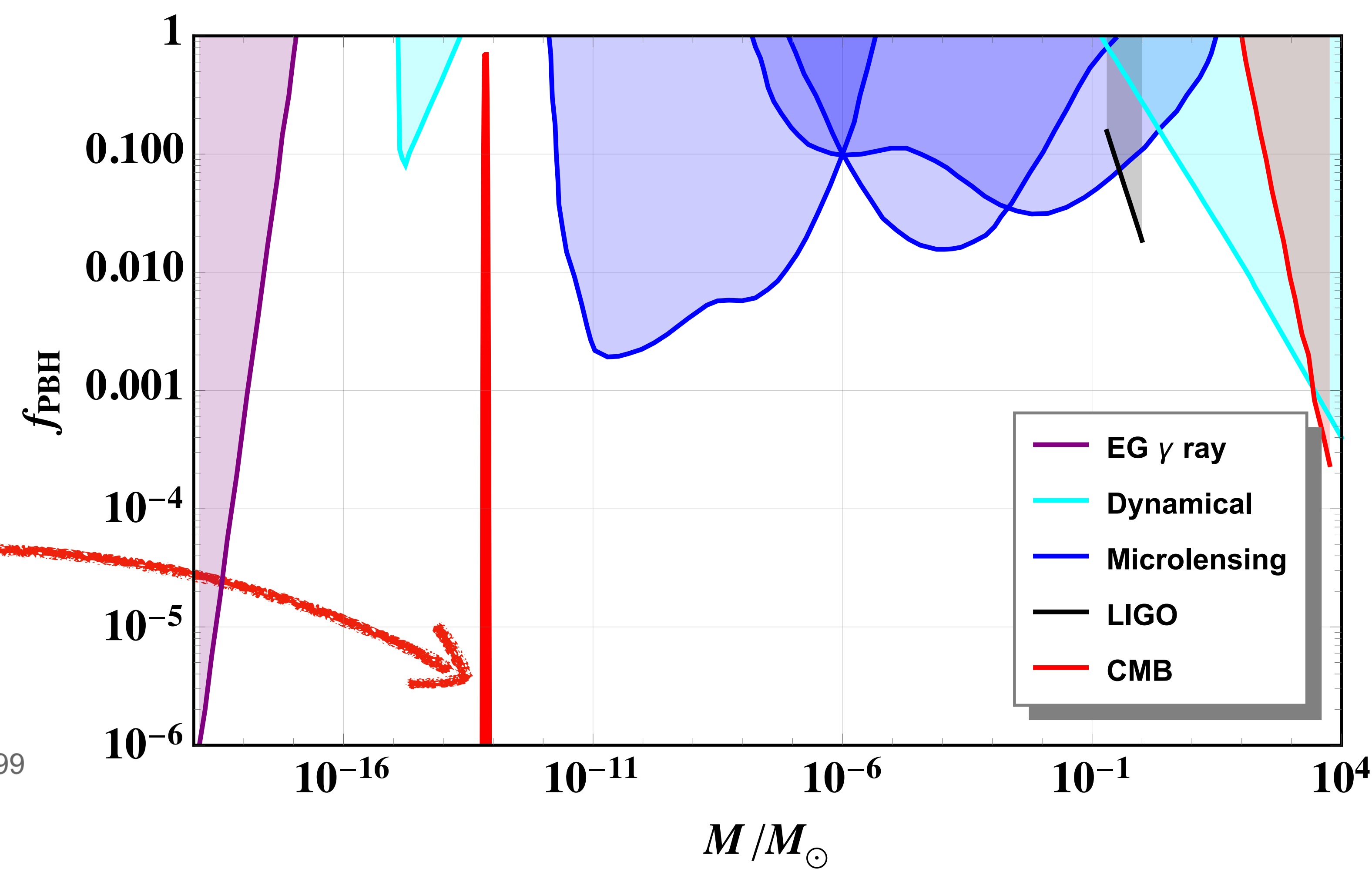
PBH from R^2-inflation

$$r = \frac{\text{tensor spectrum}}{\text{scalar spectrum}}$$



n_s = spectral tilt

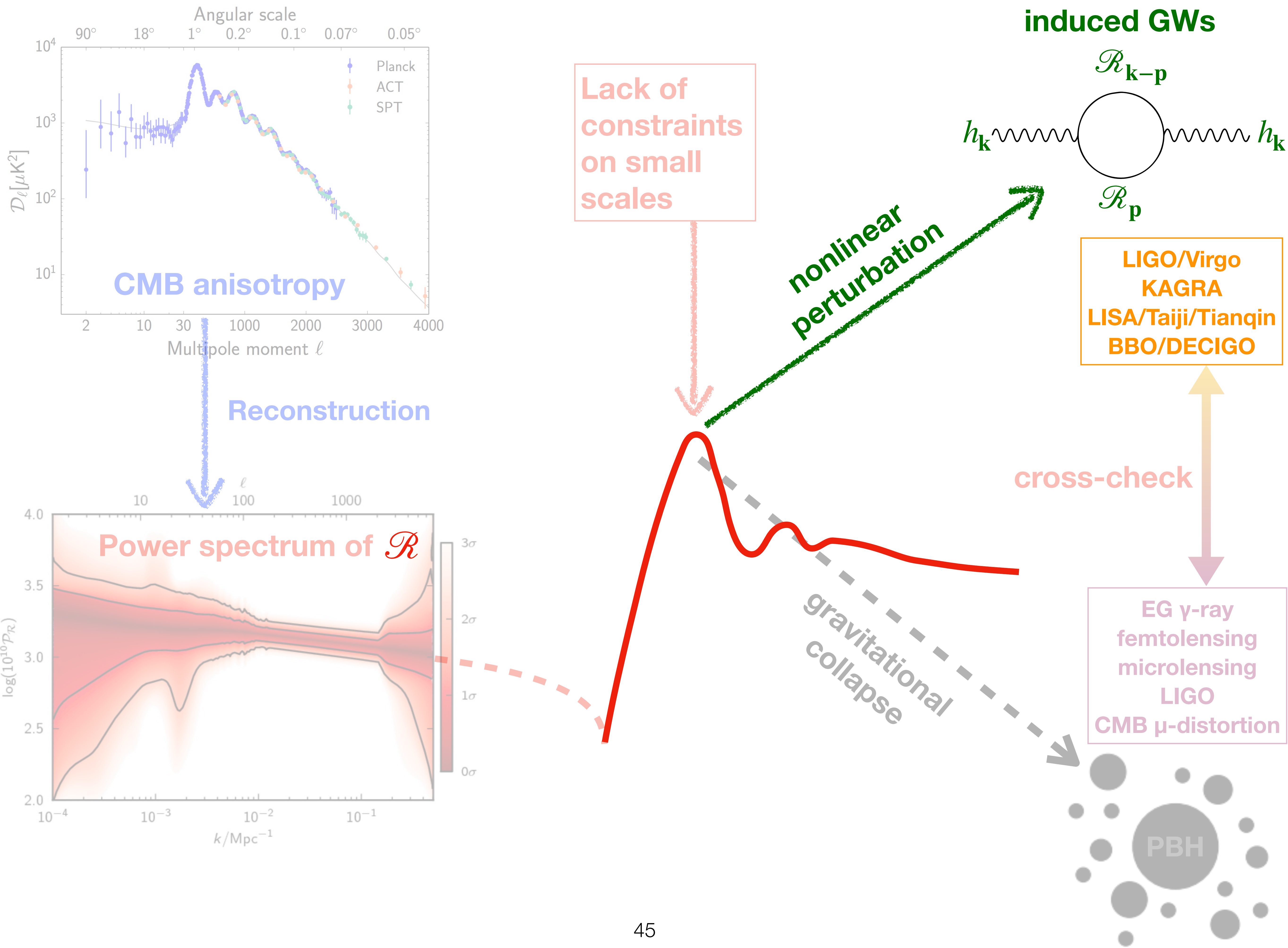
$$f_{\rm PBH} = \frac{\text{PBH energy density}}{\text{CDM energy density}}$$



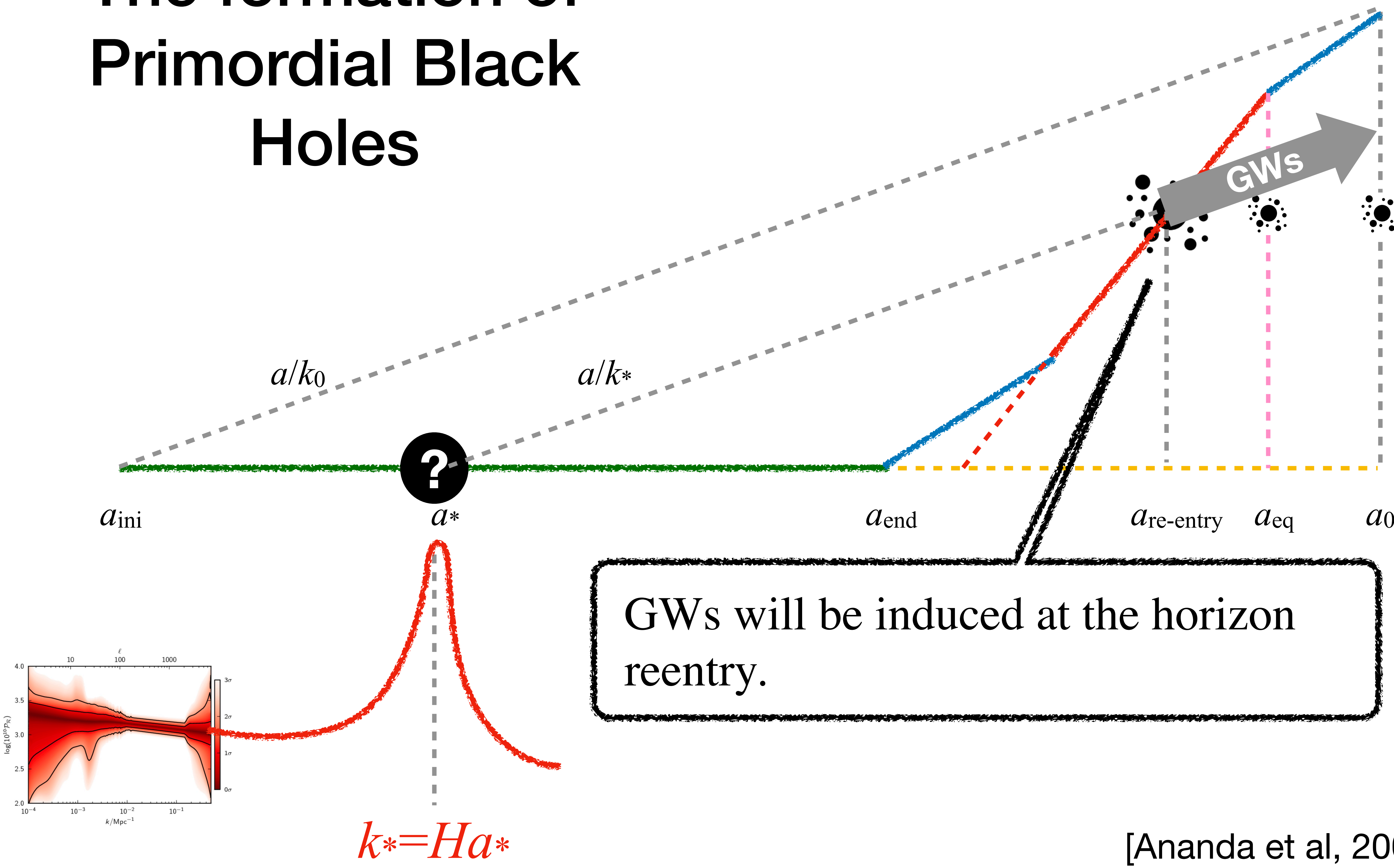
[SP, Zhang, Huang & Sasaki, JCAP1805, 042]

Content

- Introduction
- Primordial perturbation and PBHs
- Induced GWs, and their relation to PBHs
- Some properties for IGW: scaling and peak
- Summary


Angular scale
90° 18° 1° 0.2° 0.1° 0.07° 0.05°
Planck
ACT
SPT
$\mathcal{D}_\ell [\mu K^2]$
Multipole moment ℓ
CMB anisotropy
Reconstruction
Power spectrum of $\mathscr{R}$
$\log(10^{10}\mathcal{P}_\mathcal{R})$
k/Mpc^{-1}
Lack of constraints on small scales
nonlinear perturbation
gravitational collapse
induced GWs
$\mathscr{R}_{\mathbf{k-p}}$
$h_{\mathbf{k}}$
$h_{\mathbf{k}}$
$\mathscr{R}_{\mathbf{p}}$
LIGO/Virgo
KAGRA
LISA/Taiji/Tianqin
BBO/DECIGO
cross-check
EG γ-ray
femtolensing
microlensing
LIGO
CMB μ-distortion
PBH

The formation of Primordial Black Holes



GWs will be induced at the horizon reentry.

[Ananda et al, 2007]
[Baumann et al, 2007]

Induced GWs

- The metric is [Baumann+ 2007]

$$ds^2 = a(\eta)^2 \left[-(1-2\Phi)\, d\eta^2 + \left(1 + 2\Phi + h_{ij}\right) dx^i dx^j \right].$$

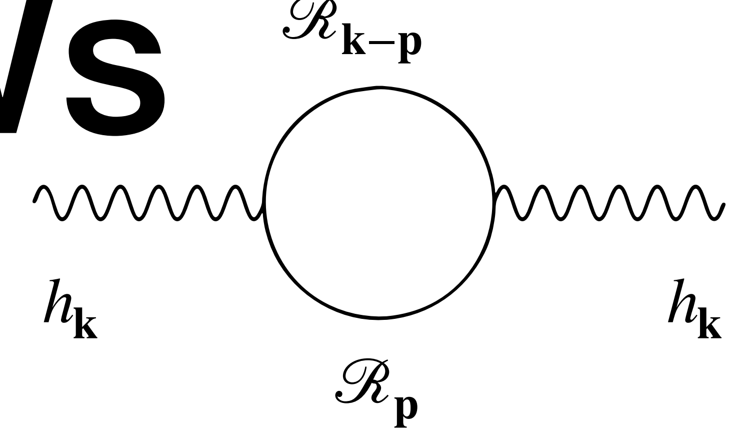
- From the nonlinear equation of motion for the tensor perturbation

$$h''_{\mathbf{k}} + 2\mathscr{H} h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = \mathcal{S}(\mathbf{k}, \eta)$$

- where the source term is

$$\mathcal{S}(\mathbf{k}, \eta) \sim \int d^3p \Phi_{\mathbf{p}} \Phi_{\mathbf{k}-\mathbf{p}} \times (\text{transfer functions})$$

Induced GWs



- The induced (secondary) GWs are

$$h_{\mathbf{k}} \sim \int d\eta \times (\text{Green function}) \int d^3p \times (\text{Transfer function}) \times \Phi_{\mathbf{p}}\Phi_{\mathbf{k}-\mathbf{p}}\,.$$

- The energy density parameter is then

$$\Omega_{\text{GW}} \sim \langle hh \rangle \sim \langle \Phi\Phi\Phi\Phi \rangle \sim \mathscr{P}_{\Phi}^2 \sim \mathscr{P}_{\mathscr{R}}^2$$

- In the radiation dominated universe we have $\mathscr{R} = \dfrac{2}{3}\Phi$

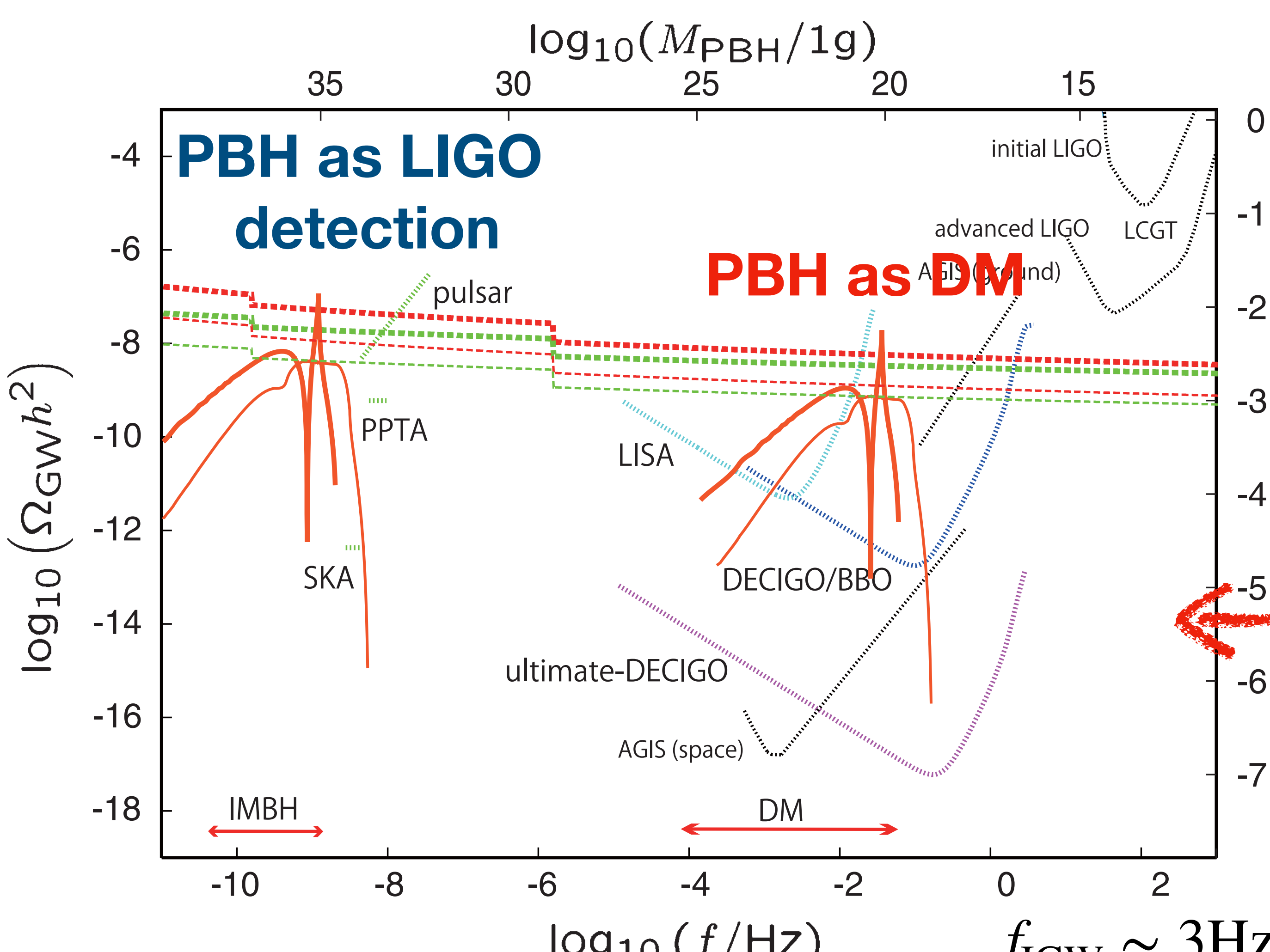
- Gauge (in)dependence: [Inomata+ 2019, Lu+ 2020]

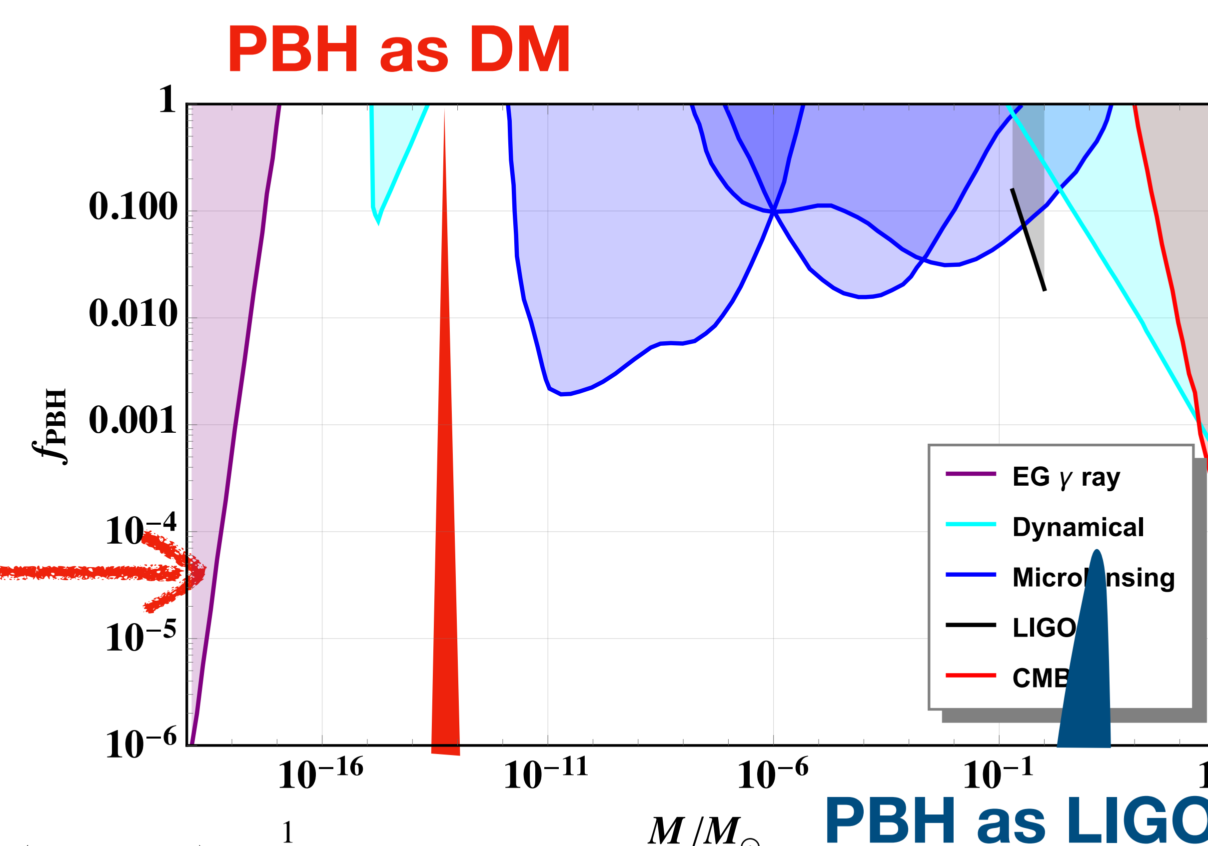
Induced GWs

$$\Omega_{\rm GW} \sim \langle hh \rangle \sim \langle \Phi\Phi\Phi\Phi \rangle \sim \mathscr{P}_\Phi^2 \sim \boxed{\mathscr{P}_\mathscr{R}^2}$$

PBH abundance

$$f_{\rm PBH} \sim 4.11 \times 10^{-8} \beta(M) \left(\frac{M}{M_\odot} \right)^{-1/2} \qquad \beta \sim {\rm erfc}\left(\frac{\mathscr{R}_c}{2\boxed{\mathscr{P}_\mathscr{R}^{1/2}}} \right)$$





PBH as LIGO detection

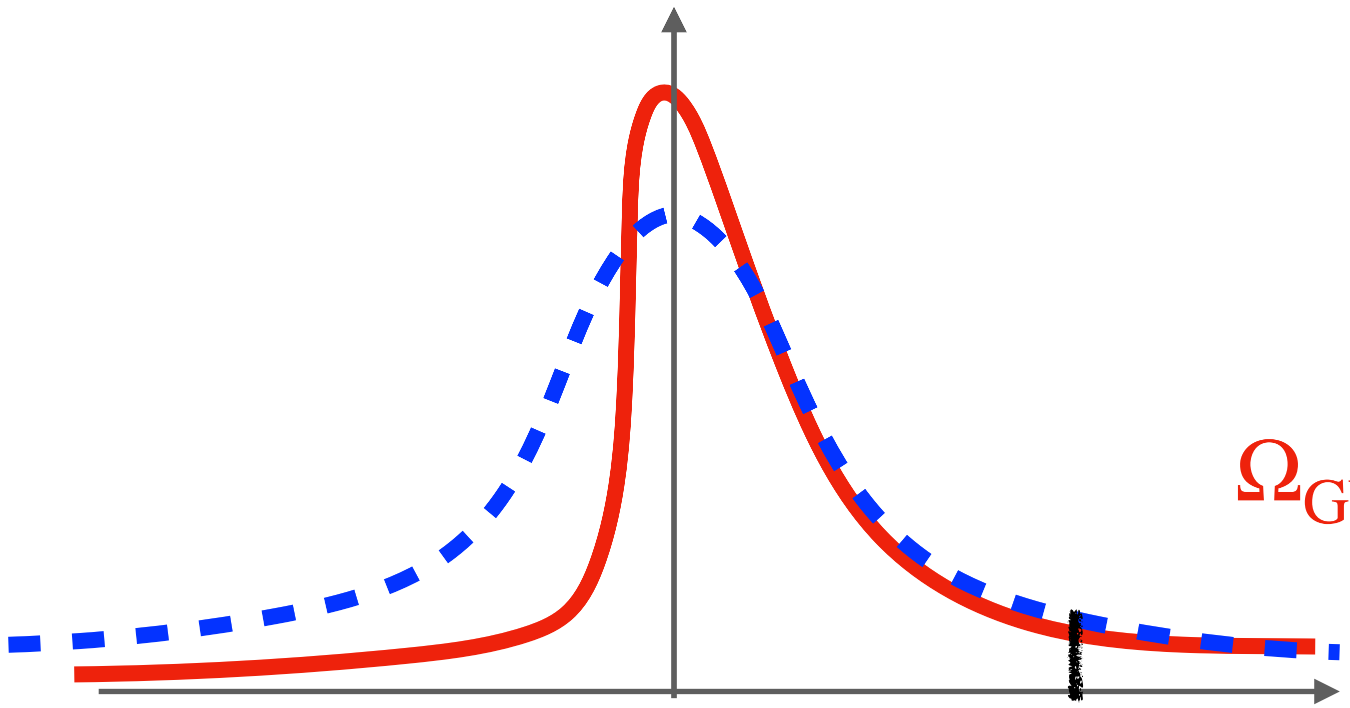
$$f_{\rm IGW} \sim 3{\rm Hz} \left(\frac{M_{\rm PBH}}{10^{16}{\rm g}} \right)^{-\frac{1}{2}}$$

[Saito and Yokoyama, 2008]

[Bird et al 2016, Sasaki et al, 2016]

PBH as DM with nG

Gaussian: $\Omega_{\rm GW} \sim \mathcal{P}^2_{\mathcal{R}}\,.$ $\beta \sim \mathrm{erfc}\left(\dfrac{\mathcal{R}_c}{2\mathcal{P}^{1/2}_{\mathcal{R}}}\right)$



Non-Gaussian:

$$\mathcal{R} = \mathcal{R}_G + F_{NL}\mathcal{R}_G^2$$

$$\Omega_{\rm GW} \sim \mathcal{P}^2_{\mathcal{R},\rm G} + F^2_{\rm NL}\mathcal{P}^3_{\mathcal{R},\rm G} + F^4_{\rm NL}\mathcal{P}^4_{\mathcal{R},\rm G}$$

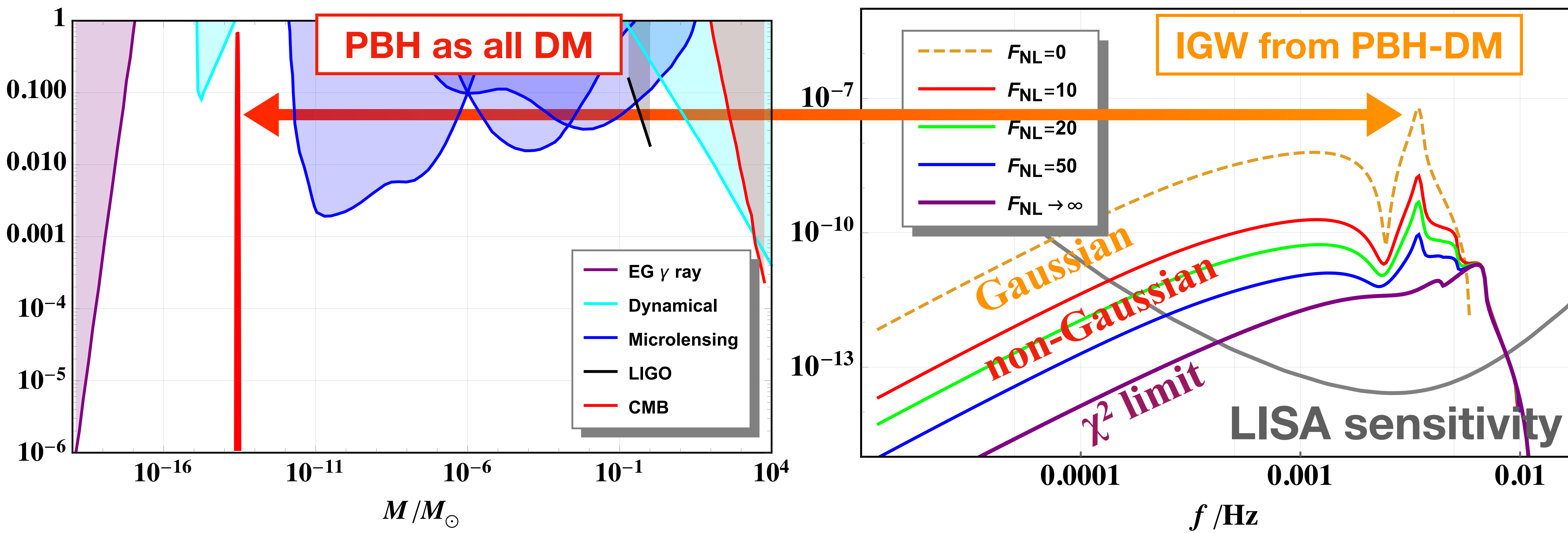
$$\mathcal{R}_{g\pm}(\mathcal{R}_c) = \frac{1}{2F_{\rm NL}}\left(-1 \pm \sqrt{1 + 4F_{\rm NL}\left(F_{\rm NL}\mathcal{P}_{\mathcal{R},G} + \mathcal{R}_c\right)}\right).$$

$$\beta = \frac{1}{2}\mathrm{erfc}\left(\frac{\mathcal{R}_{g+}(\mathcal{R}_c)}{\sqrt{2\mathcal{P}_{\mathcal{R},G}}}\right) + \frac{1}{2}\mathrm{erfc}\left(-\frac{\mathcal{R}_{g-}(\mathcal{R}_c)}{\sqrt{2\mathcal{P}_{\mathcal{R},G}}}\right); \quad f_{\rm NL} > 0.$$

[Young and Byrnes 2013]

LISA to detect PBH-DM

$$f_{\mathrm{PBH}} = \frac{\text{PBH energy density}}{\text{CDM energy density}} \qquad \Omega_{\mathrm{GW}} = \frac{\text{GW energy}}{\text{critical energy density}}$$



[Cai, SP, and Sasaki, Phys.Rev.Lett.122, 201101]

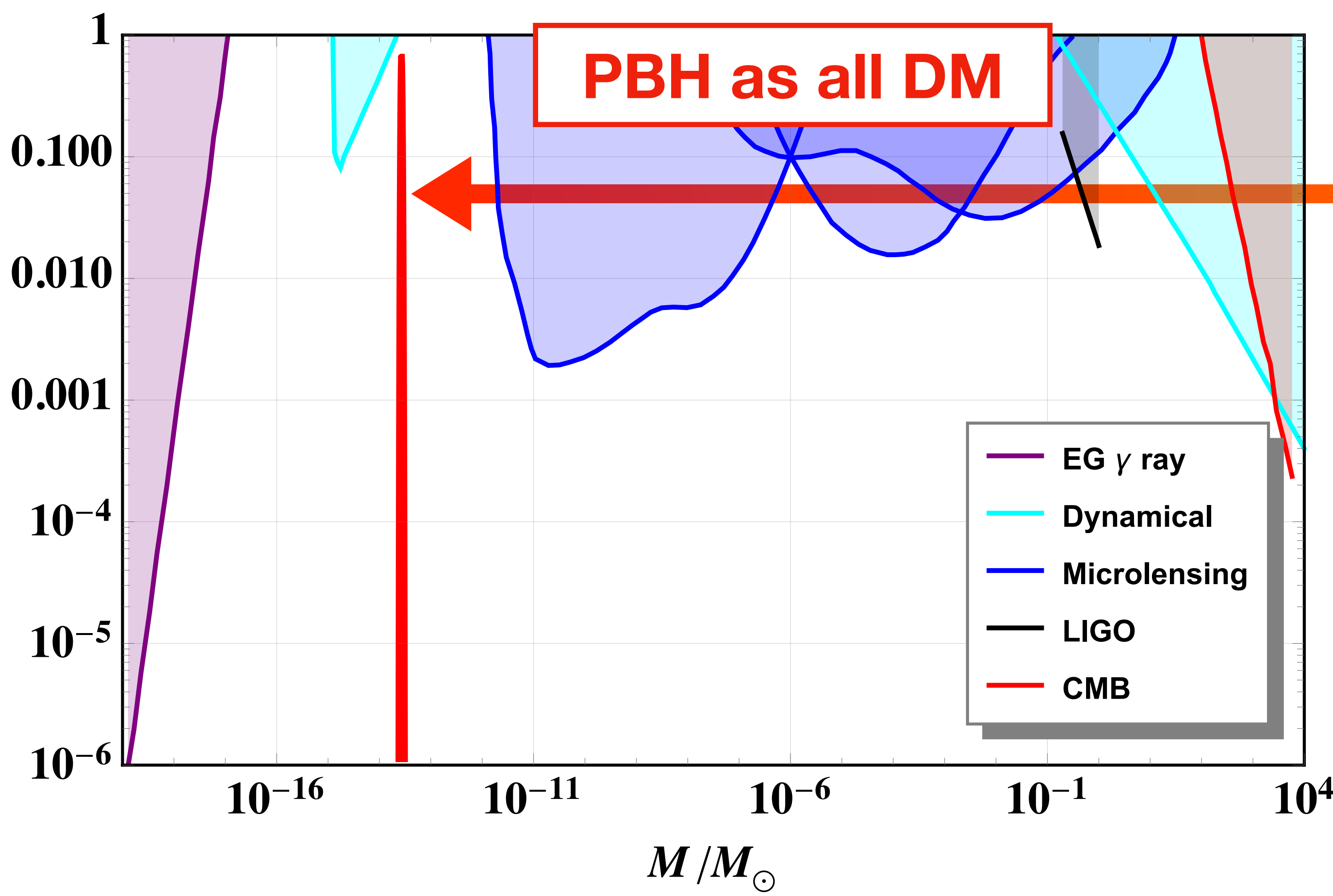
If PBH serves as all DM, the induced GWs must be detectable by LISA, independent of $\mathcal{A}_{\mathcal{R}}$ or $F_{\rm NL}$.

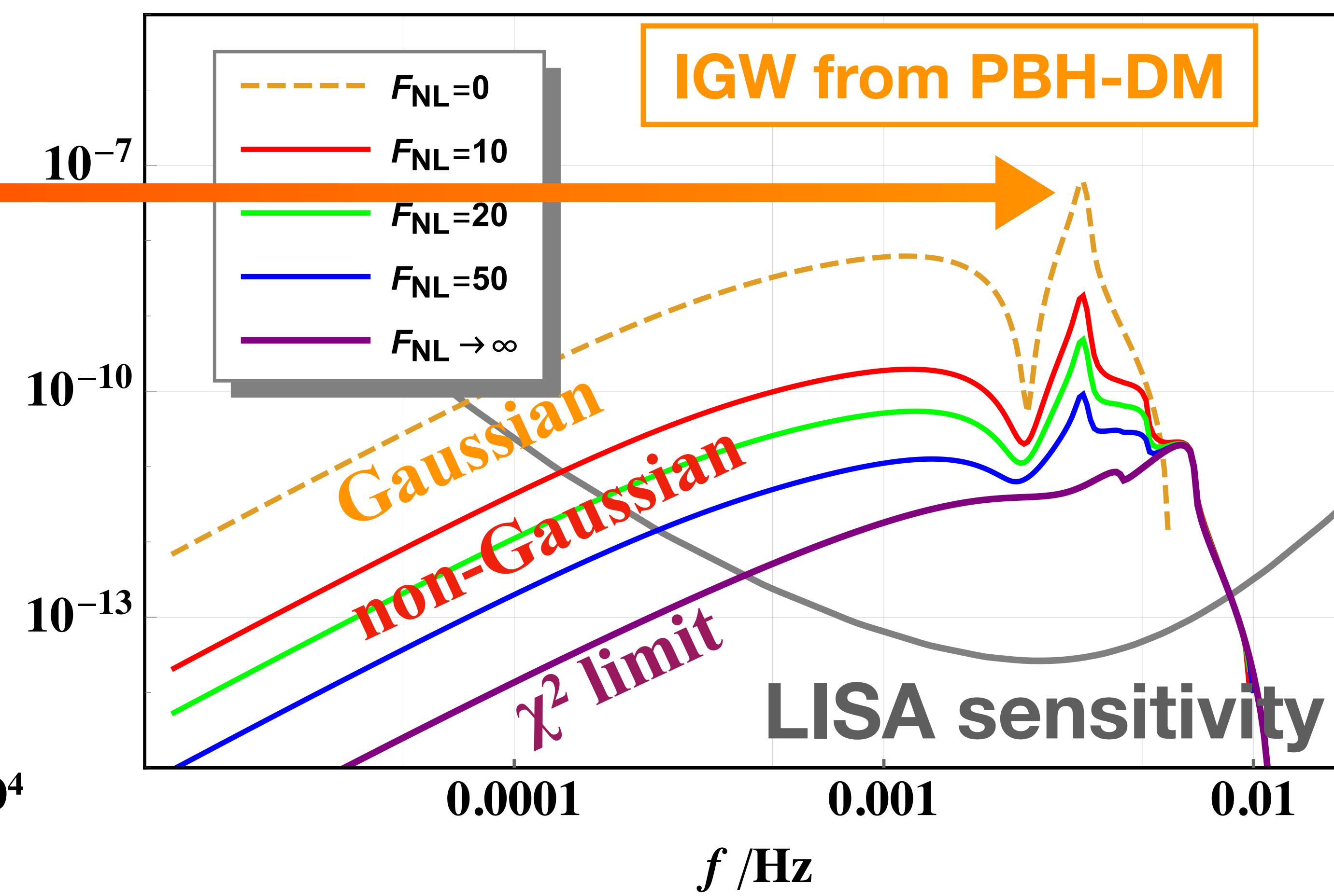
$$\mathcal{R} = \mathcal{R}_G + F_{NL}\mathcal{R}_G^2$$

$$\Omega_{\rm GW} \sim \mathcal{P}^2_{\mathcal{R},\rm G} + F_{\rm NL}^2\mathcal{P}^3_{\mathcal{R},\rm G} + F_{\rm NL}^4\mathcal{P}^4_{\mathcal{R},\rm G}$$

$$f_{\rm PBH} = \frac{\text{PBH energy density}}{\text{CDM energy density}}$$

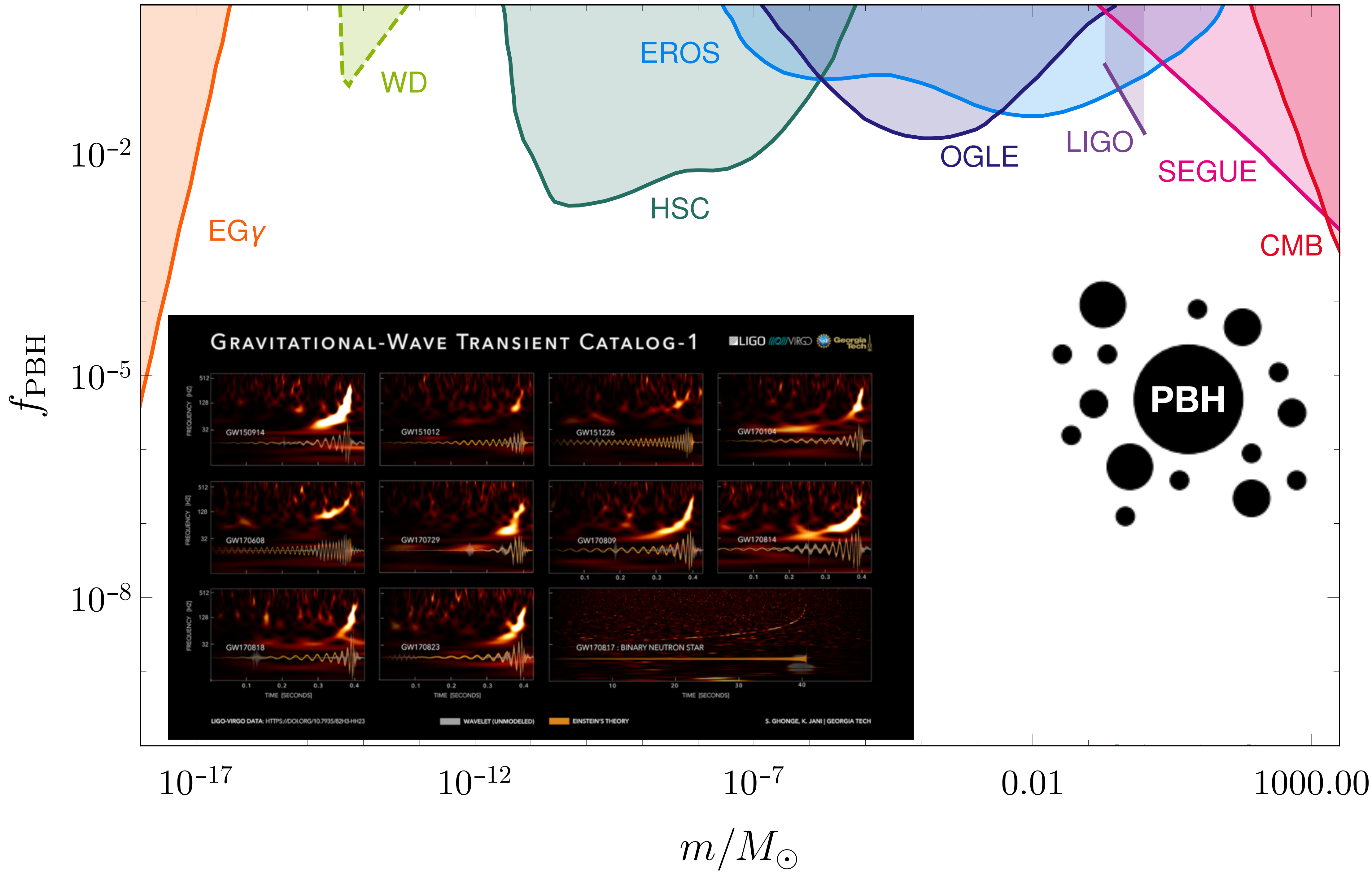
$$\Omega_{\rm GW} = \frac{\text{GW energy}}{\text{critical energy density}}$$





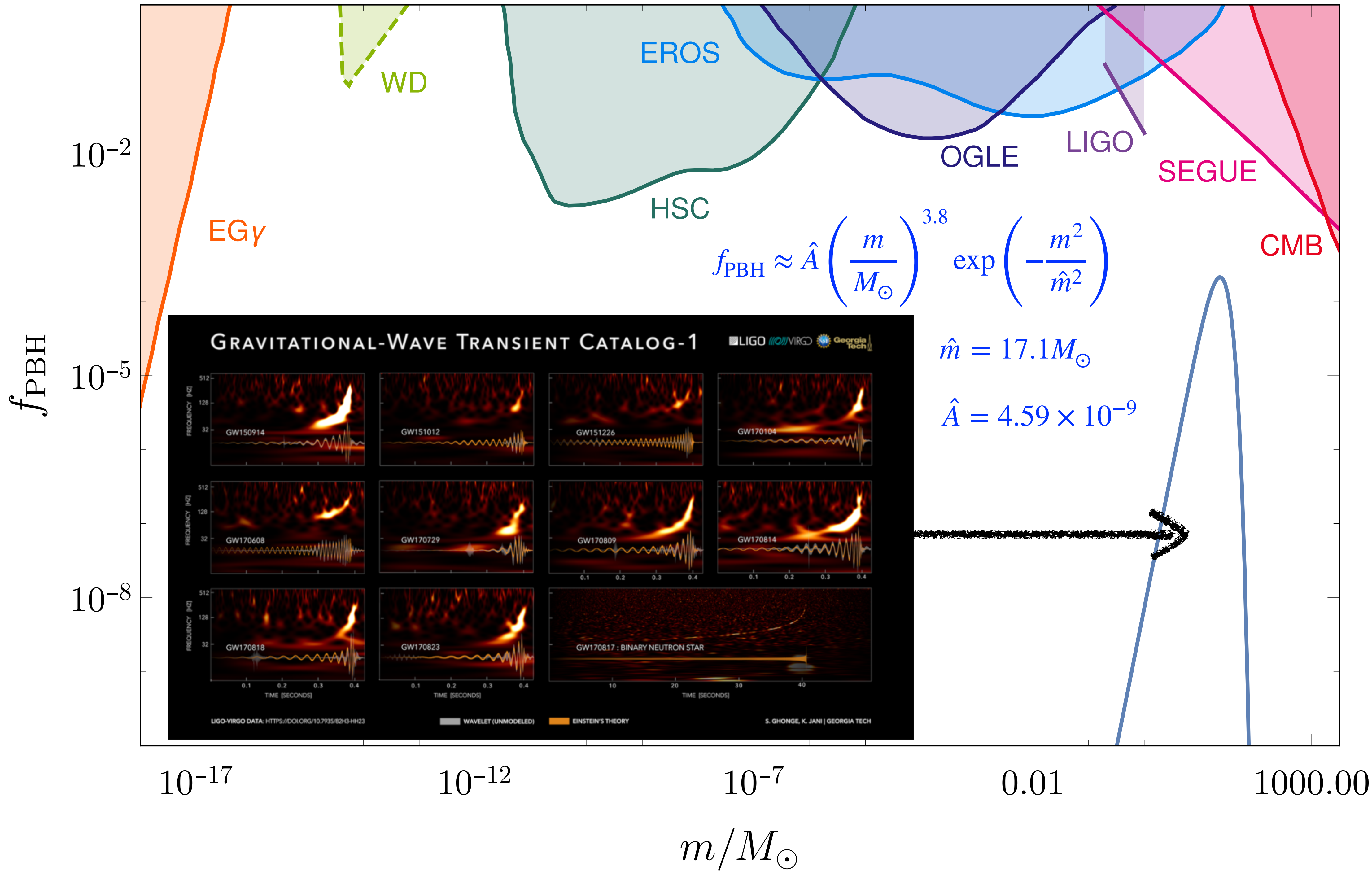
[Cai, SP, and Sasaki, Phys.Rev.Lett.122, 201101]

PBHs as LIGO events



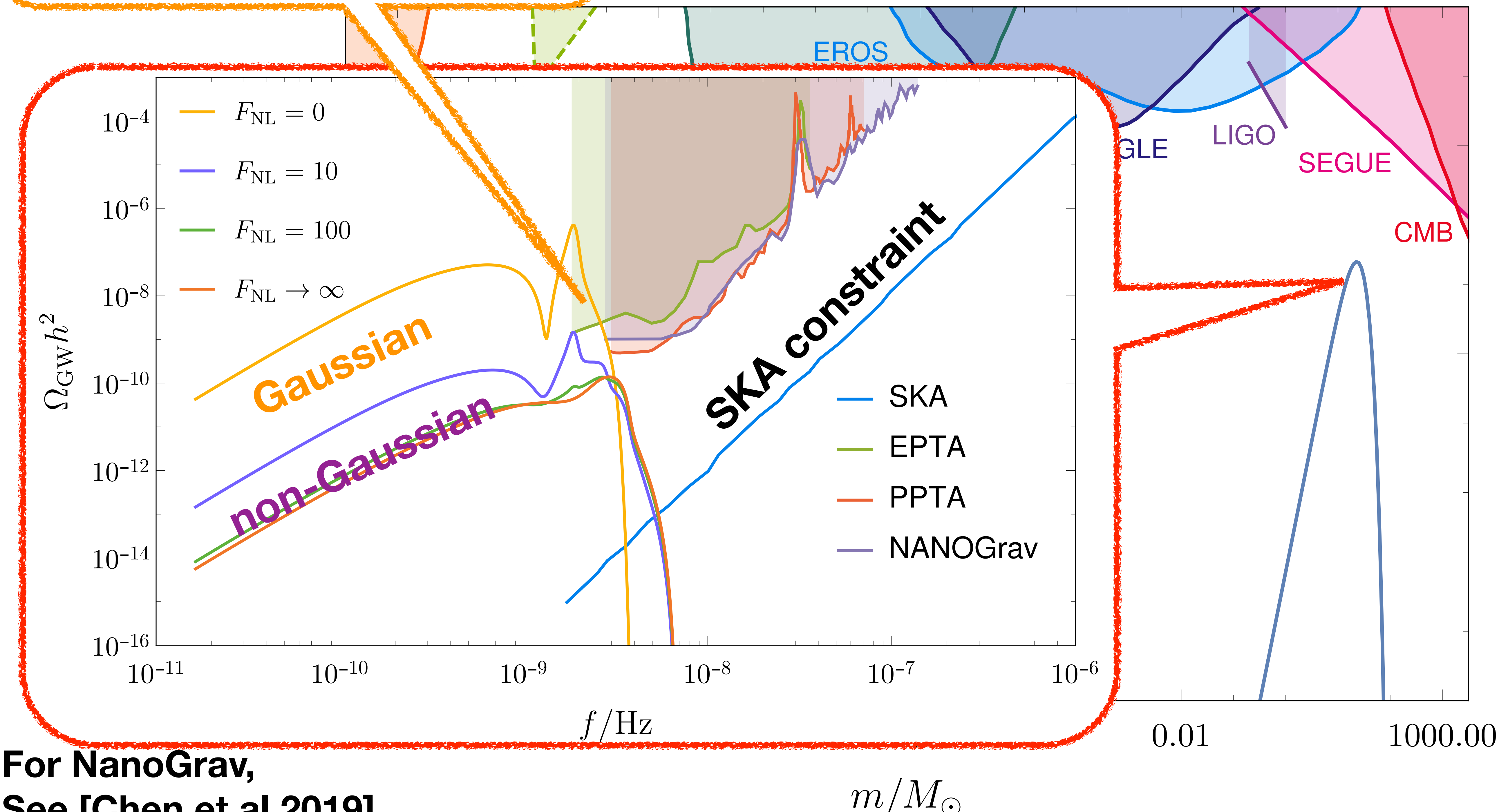
[Cai, SP, Wang, and Yang, JCAP1910, 059]

PBHs as LIGO events



[Cai, SP, Wang, and Yang, JCAP1910, 059]

The IGW from LIGO-PBH is excluded by PTA…

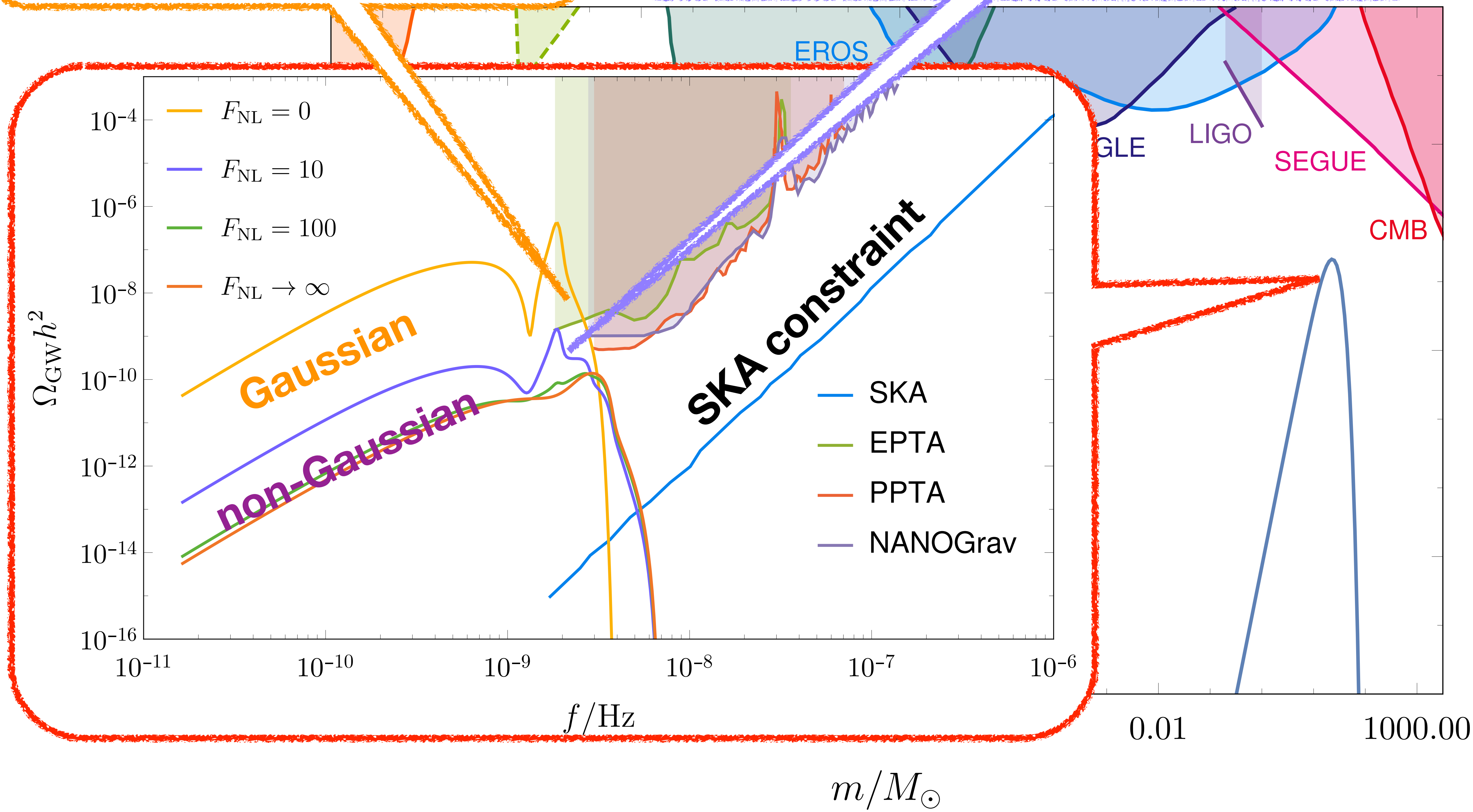


For NanoGrav,
See [Chen et al 2019]
Stronger constraints,
but smaller mass

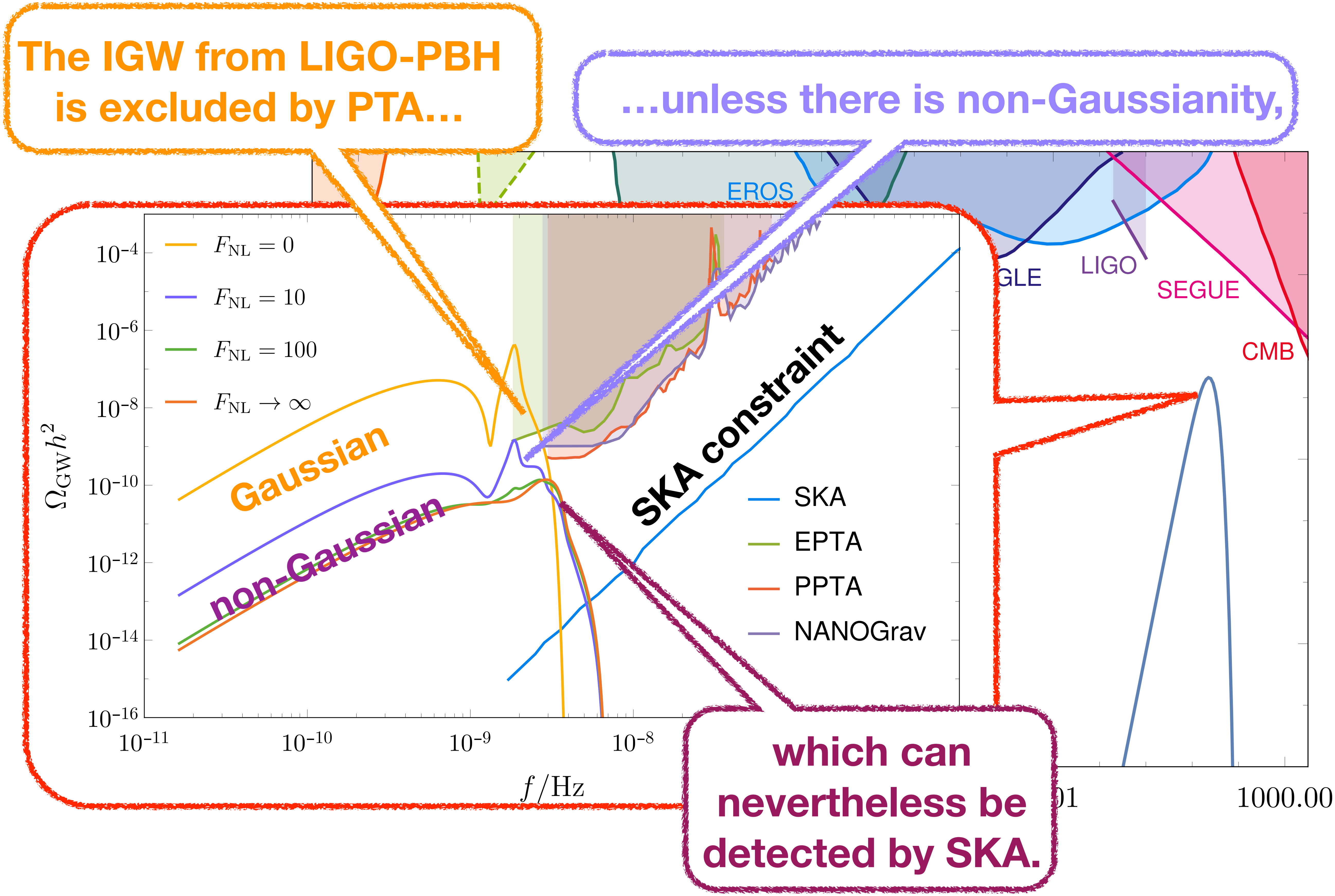
[Cai, SP, Wang, and Yang, JCAP1910, 059]

The IGW from LIGO-PBH is excluded by PTA…

…unless there is non-Gaussianity,



[Cai, SP, Wang, and Yang, JCAP1910, 059]

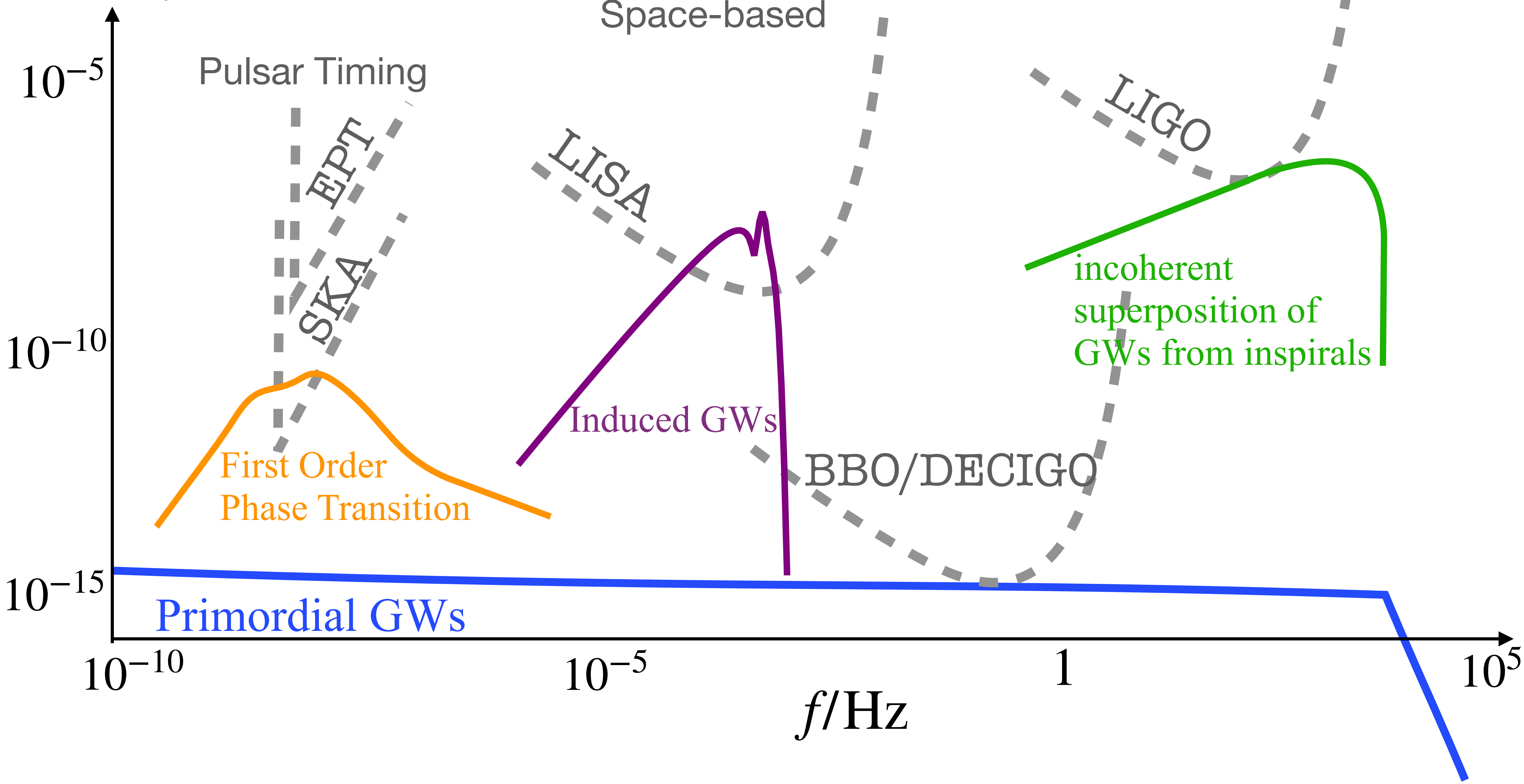


[Cai, SP, Wang, and Yang, JCAP1910, 059]

Content

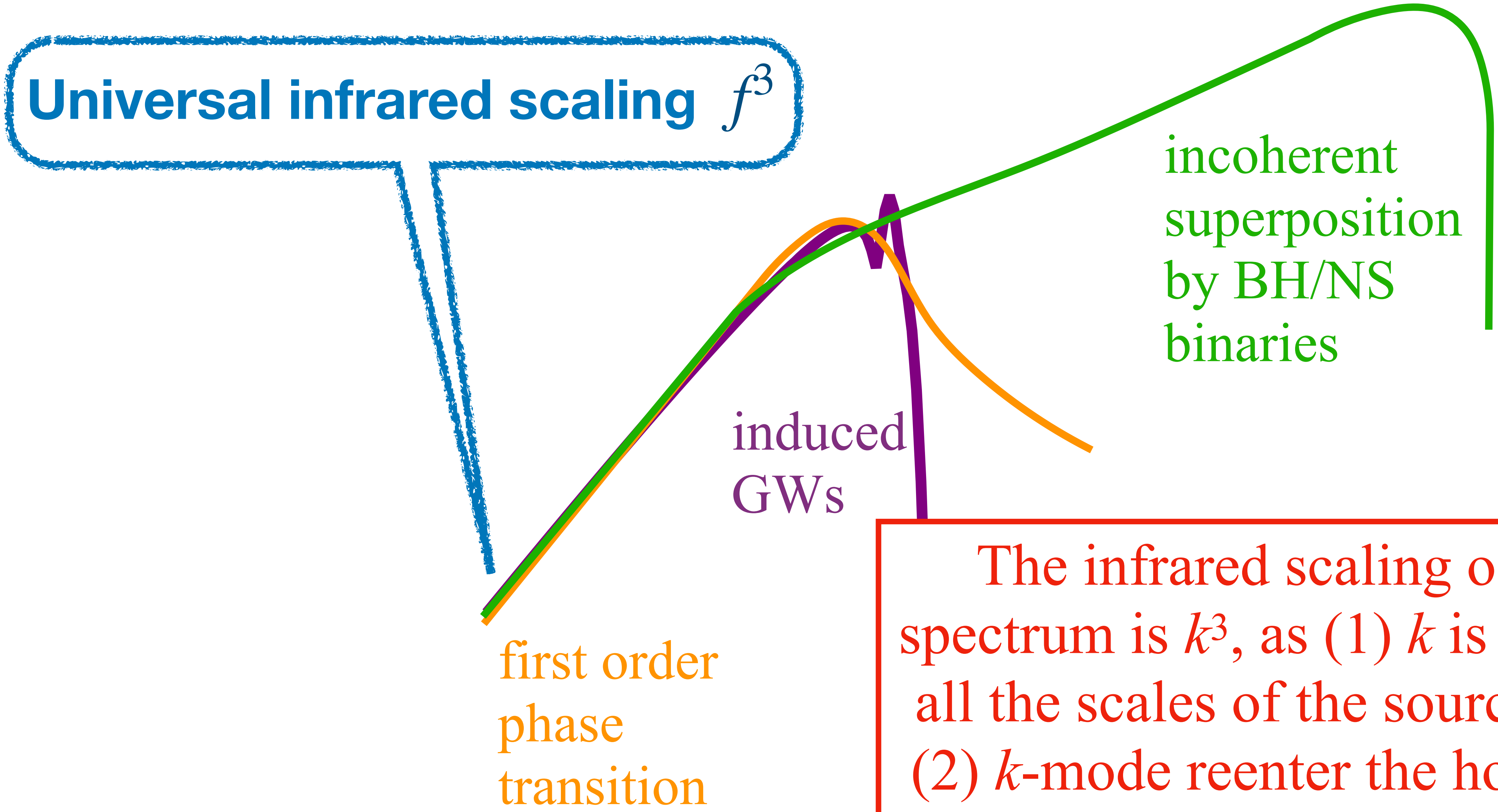
Infrared Scaling of SGWB

$$\Omega_{\mathrm{GW}} \equiv \frac{1}{\rho_{\mathrm{cr}}} \frac{d\rho_{\mathrm{GW}}}{d \log k}$$



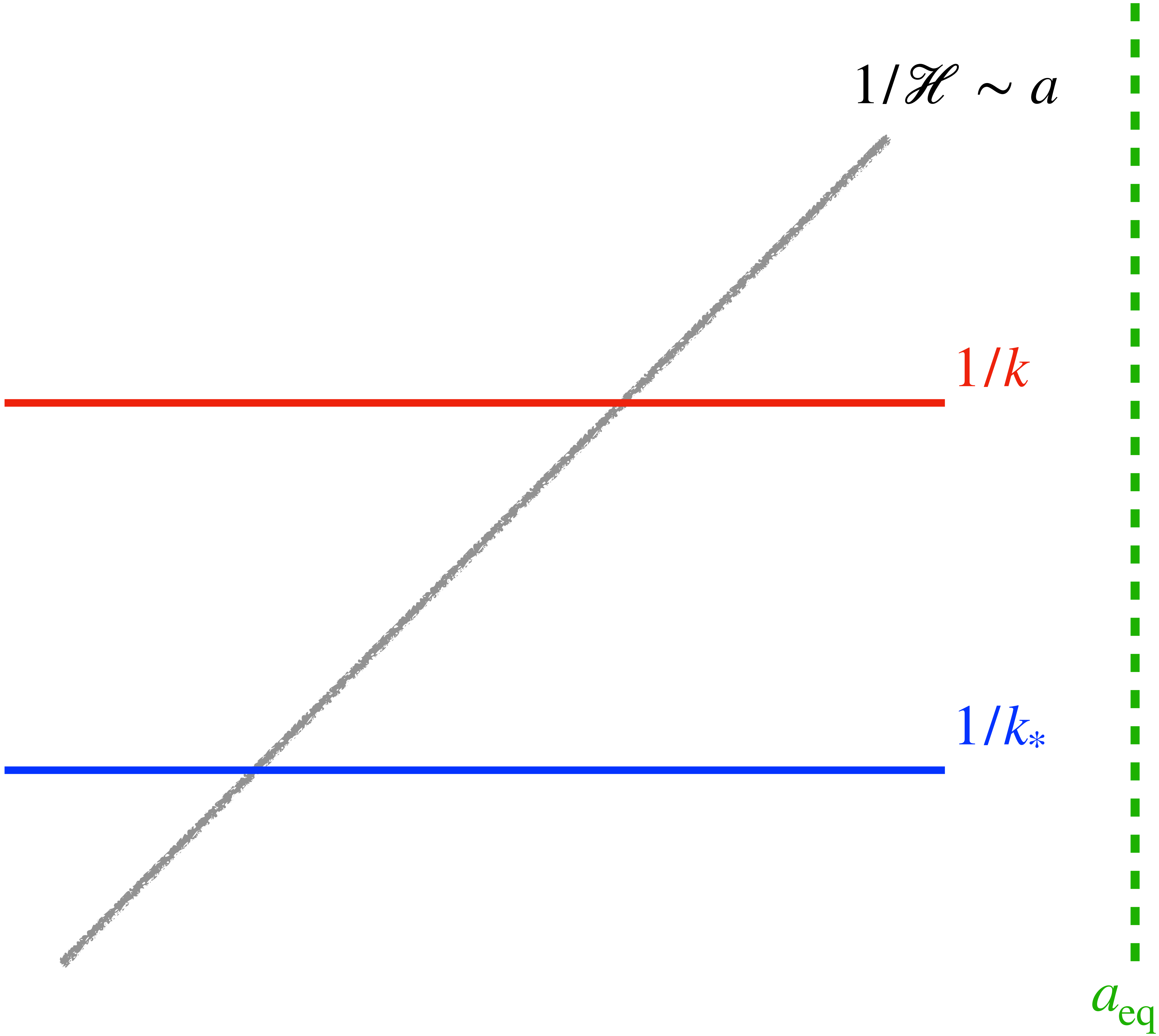
[Cai, SP, and Sasaki, arXiv:1909.13728]

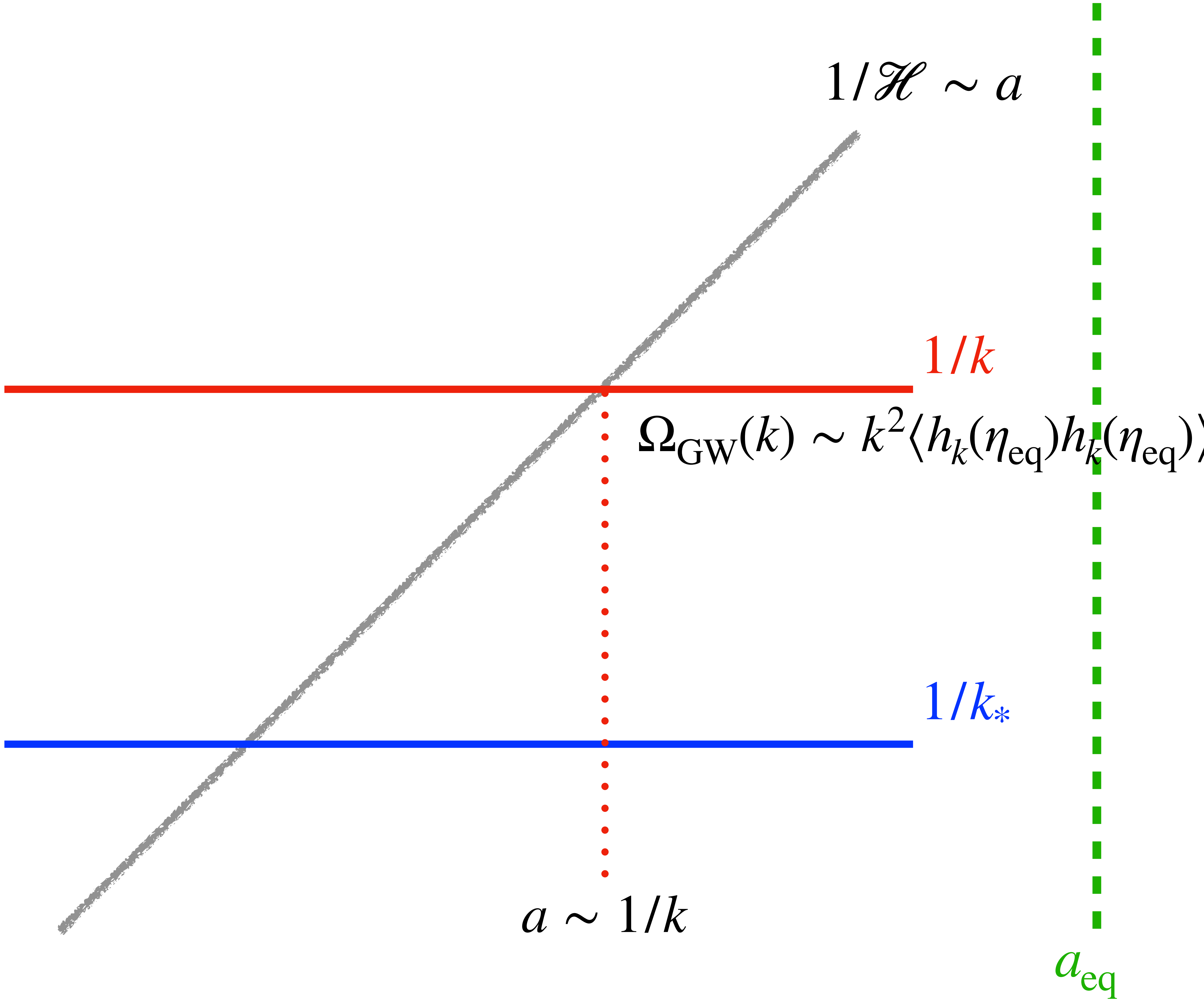
Infrared Scaling of SGWB



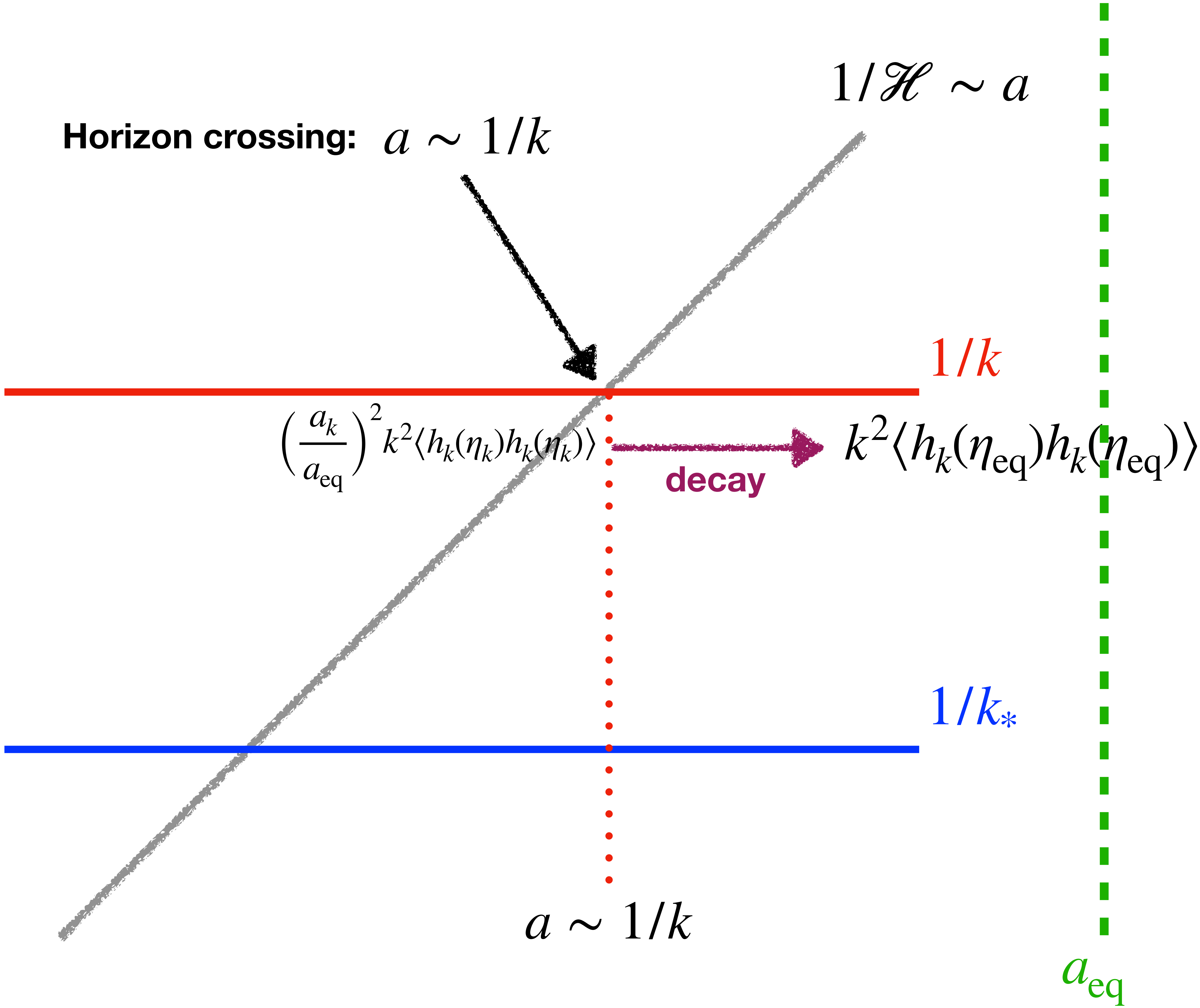
The infrared scaling of the GW spectrum is k^3, as (1) k is smaller than all the scales of the source term; and (2) k-mode reenter the horizon in the radiation dominated universe.

[Cai, SP, and Sasaki, arXiv:1909.13728]

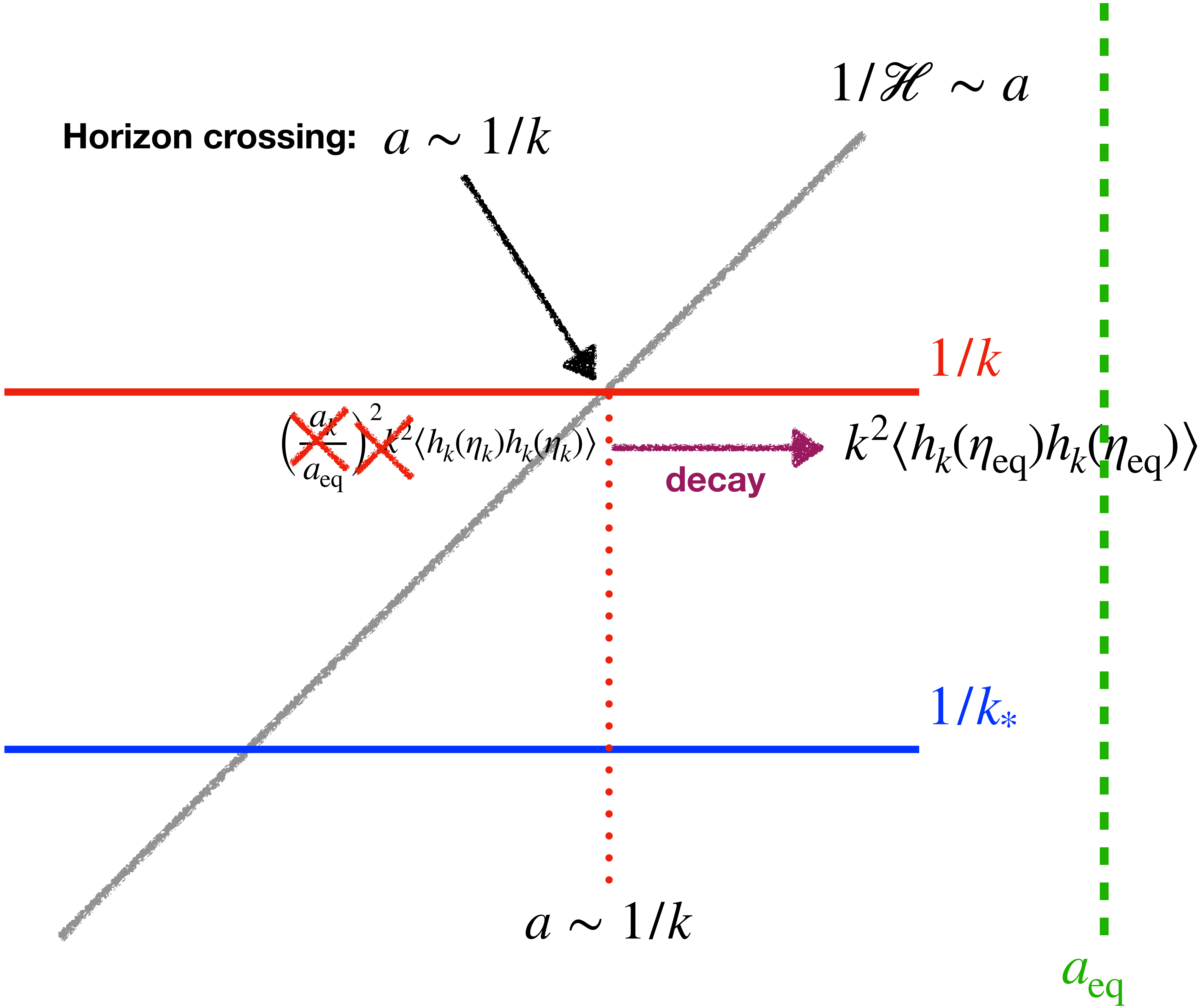

$1/\mathscr{H} \sim a$
$1/k$
$1/k_*$
$a_{\rm eq}$



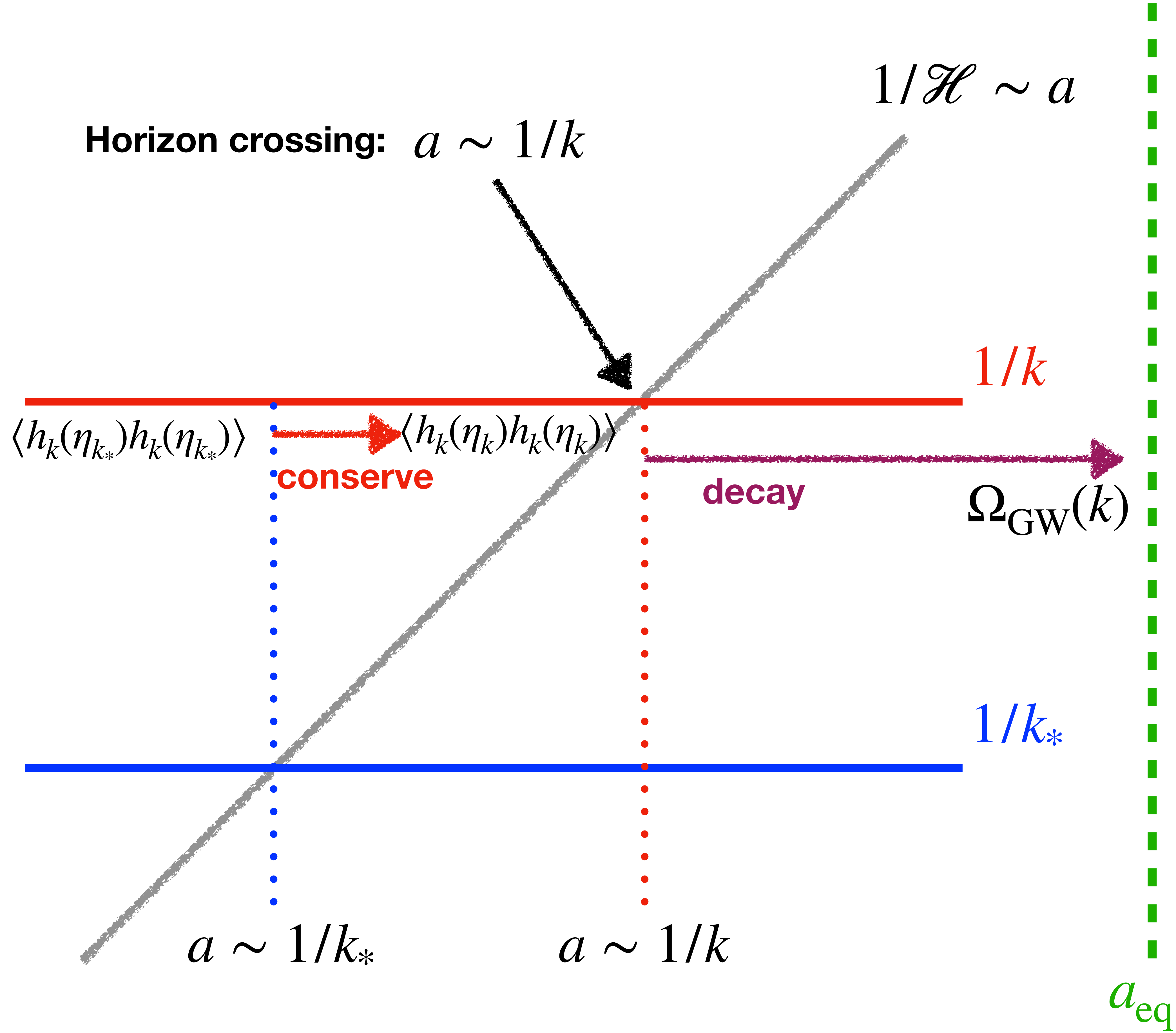
$1/\mathscr{H} \sim a$
$1/k$
$\Omega_{\mathrm{GW}}(k) \sim k^2 \langle h_k(\eta_{\mathrm{eq}}) h_k(\eta_{\mathrm{eq}}) \rangle$
$1/k_*$
$a \sim 1/k$
a_{eq}



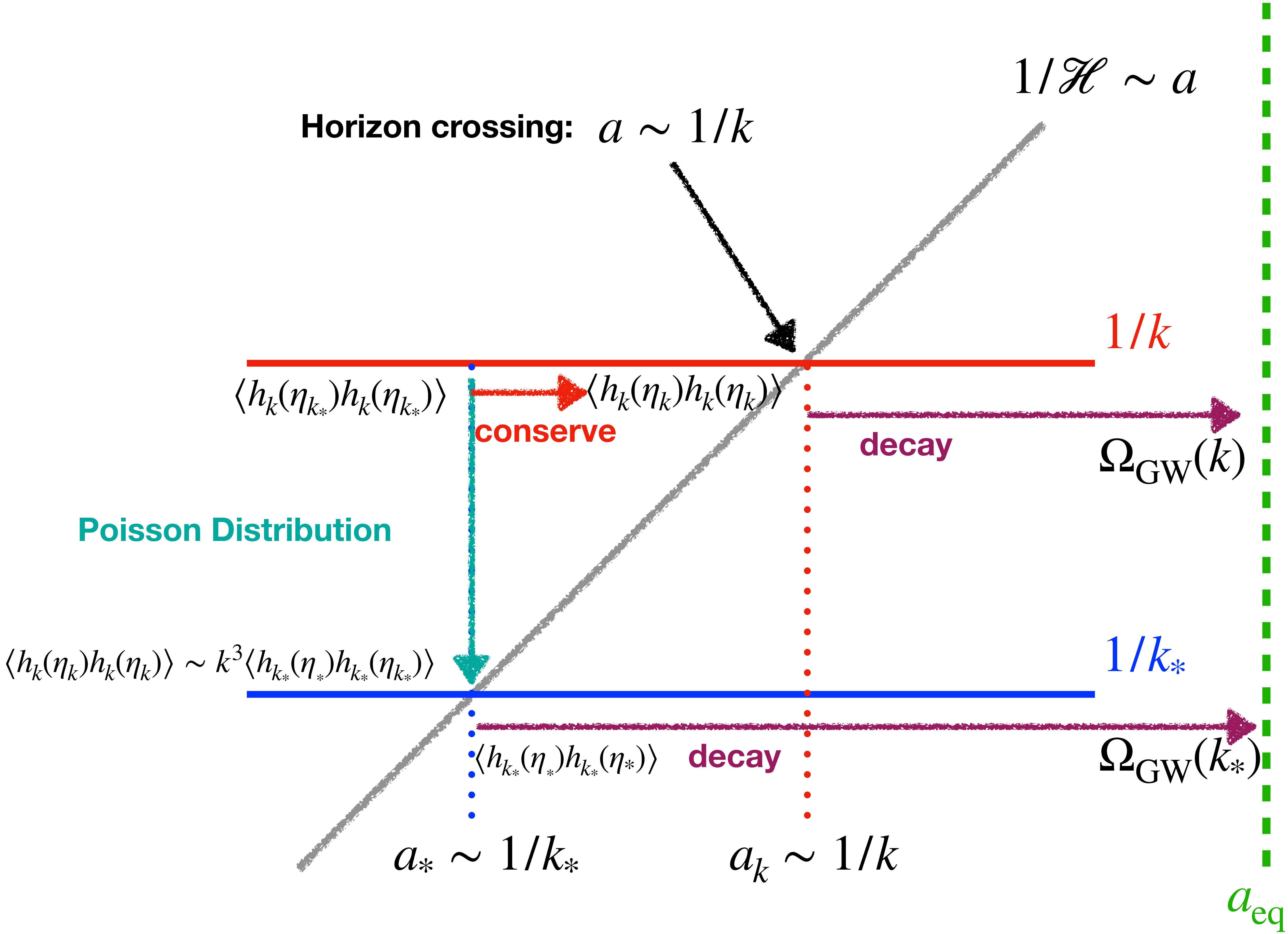
Horizon crossing: $a \sim 1/k$
$1/\mathscr{H} \sim a$
$1/k$
$\left(\frac{a_k}{a_{\rm eq}}\right)^2 k^2 \langle h_k(\eta_k) h_k(\eta_k)\rangle$
$k^2 \langle h_k(\eta_{\rm eq}) h_k(\eta_{\rm eq})\rangle$
decay
$1/k_*$
$a \sim 1/k$
$a_{\rm eq}$



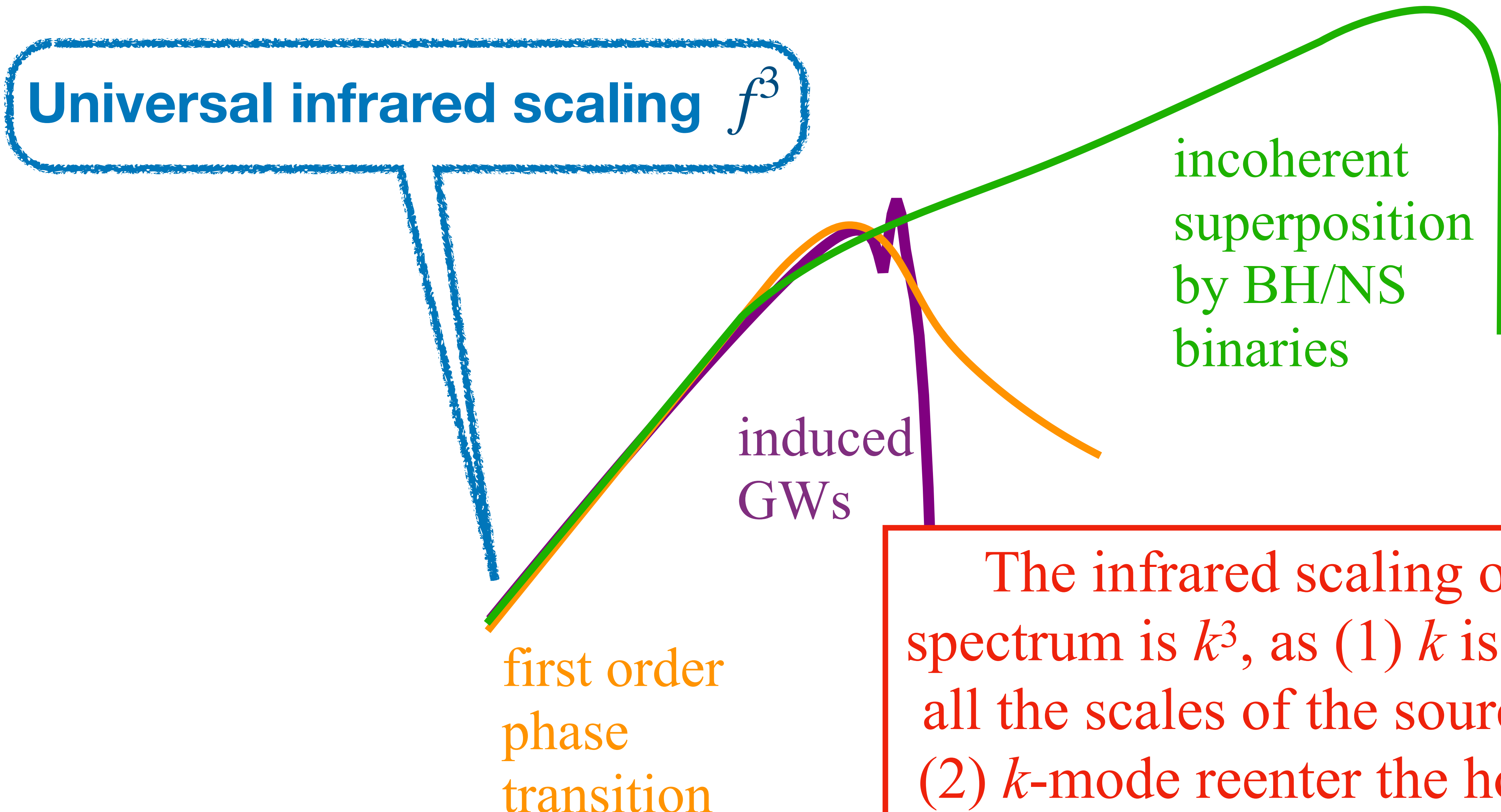
$1/\mathscr{H} \sim a$
Horizon crossing: $a \sim 1/k$
$1/k$
$\cancel{\left(\frac{a_k}{a_{eq}}\right)^2} \cancel{k^2} \langle h_k(\eta_k) h_k(\eta_k) \rangle$
decay
$k^2 \langle h_k(\eta_{eq}) h_k(\eta_{eq}) \rangle$
$1/k_*$
$a \sim 1/k$
a_{eq}



$1/\mathscr{H} \sim a$
Horizon crossing: $a \sim 1/k$
$1/k$
$\langle h_k(\eta_{k_*}) h_k(\eta_{k_*}) \rangle$
$\langle h_k(\eta_k) h_k(\eta_k) \rangle$
conserve
decay
$\Omega_{\rm GW}(k)$
$1/k_*$
$a \sim 1/k_*$
$a \sim 1/k$
$a_{\rm eq}$


$1/\mathscr{H} \sim a$
Horizon crossing: $a \sim 1/k$
$1/k$
$\langle h_k(\eta_{k_*}) h_k(\eta_{k_*})\rangle$
$\langle h_k(\eta_k) h_k(\eta_k)\rangle$
conserve
decay
$\Omega_{\rm GW}(k)$
Poisson Distribution
$\langle h_k(\eta_k) h_k(\eta_k)\rangle \sim k^3 \langle h_{k_*}(\eta_*) h_{k_*}(\eta_{k_*})\rangle$
$1/k_*$
$\langle h_{k_*}(\eta_*) h_{k_*}(\eta_*)\rangle$
decay
$\Omega_{\rm GW}(k_*)$
$a_* \sim 1/k_*$
$a_k \sim 1/k$
$a_{\rm eq}$

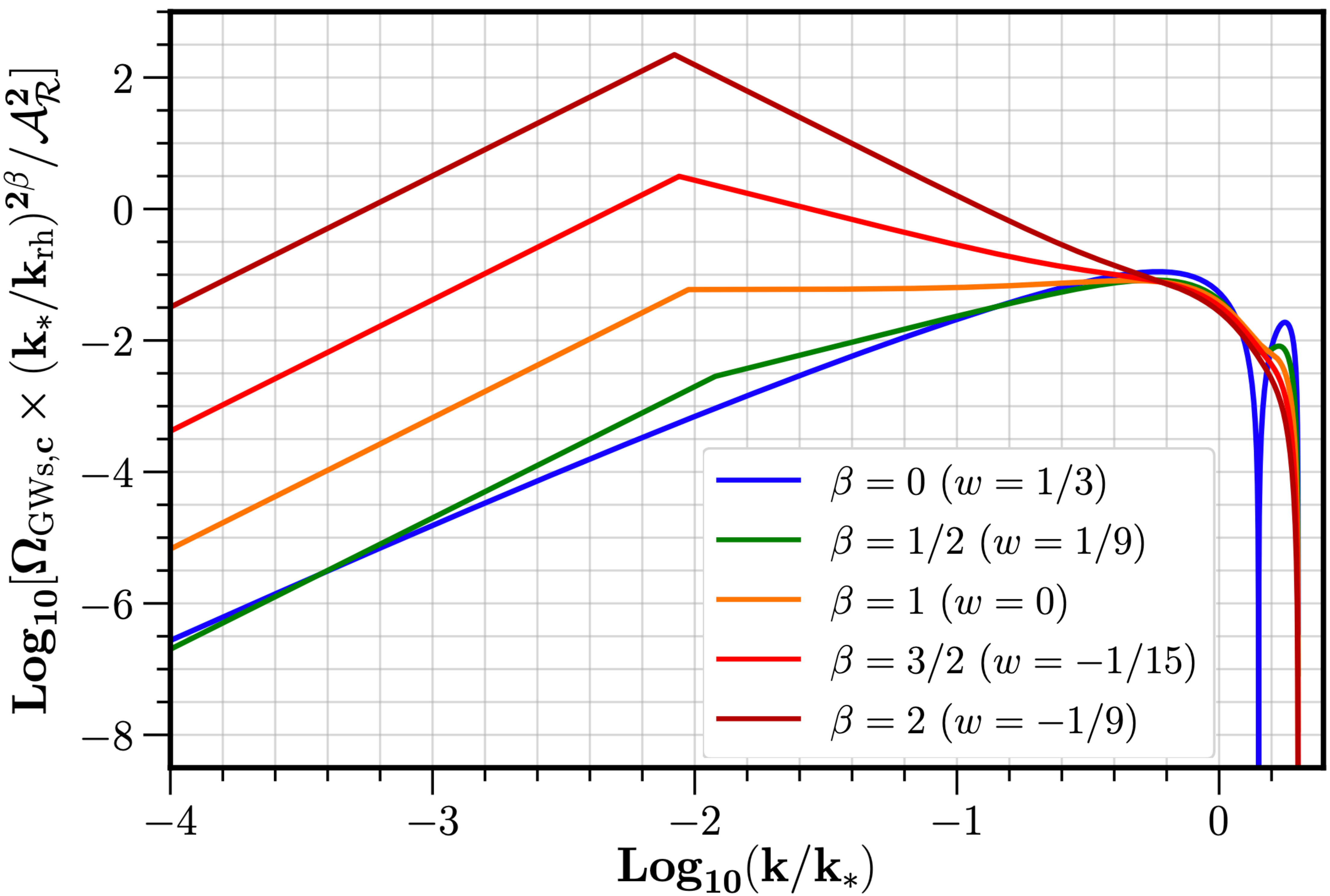
Infrared Scaling of SGWB



The infrared scaling of the GW spectrum is k^3, as (1) k is smaller than all the scales of the source term; and (2) k-mode reenter the horizon in the radiation dominated universe.

[Cai, SP, and Sasaki, arXiv:1909.13728]

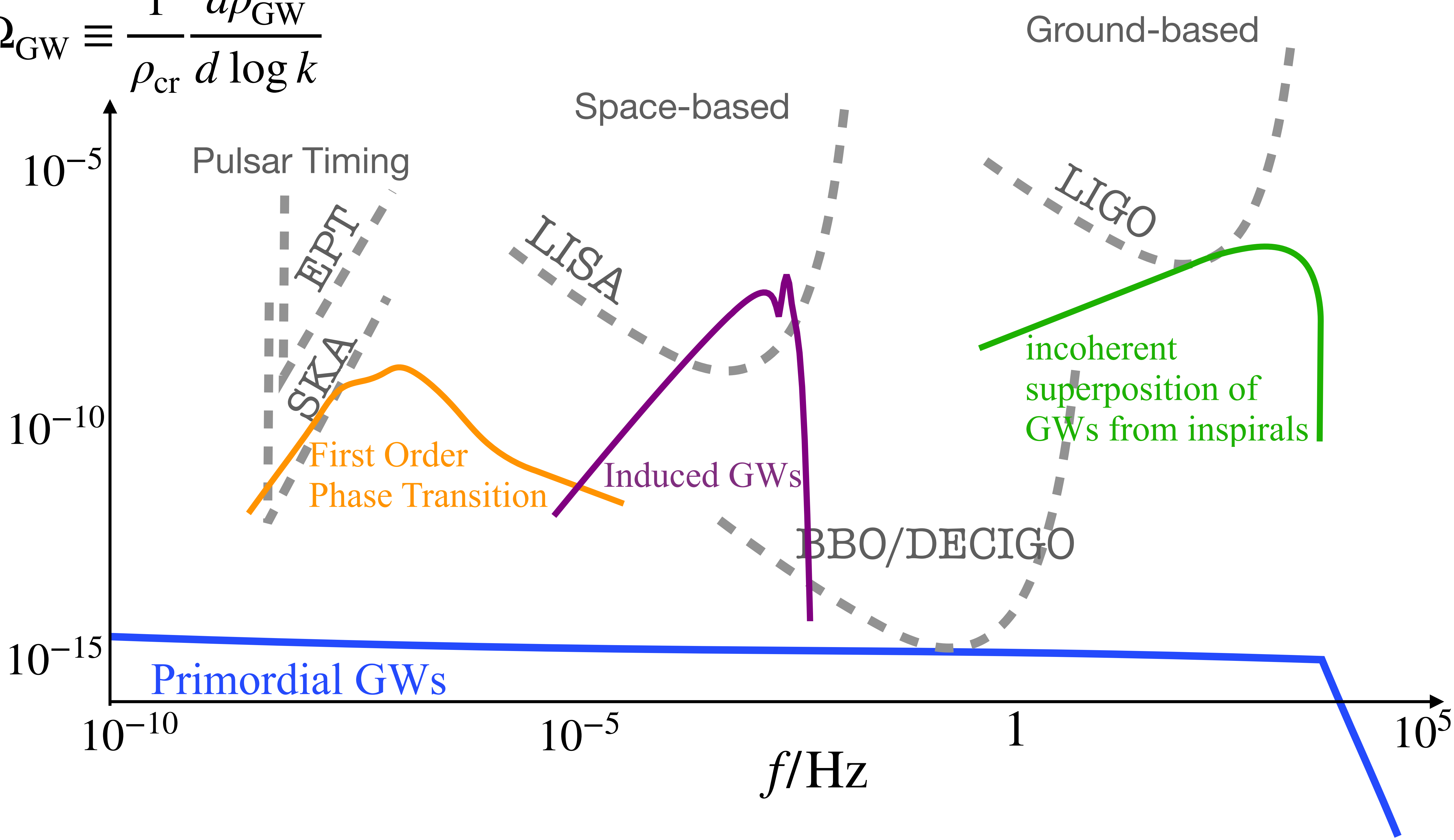
Infrared Scaling of SGWB



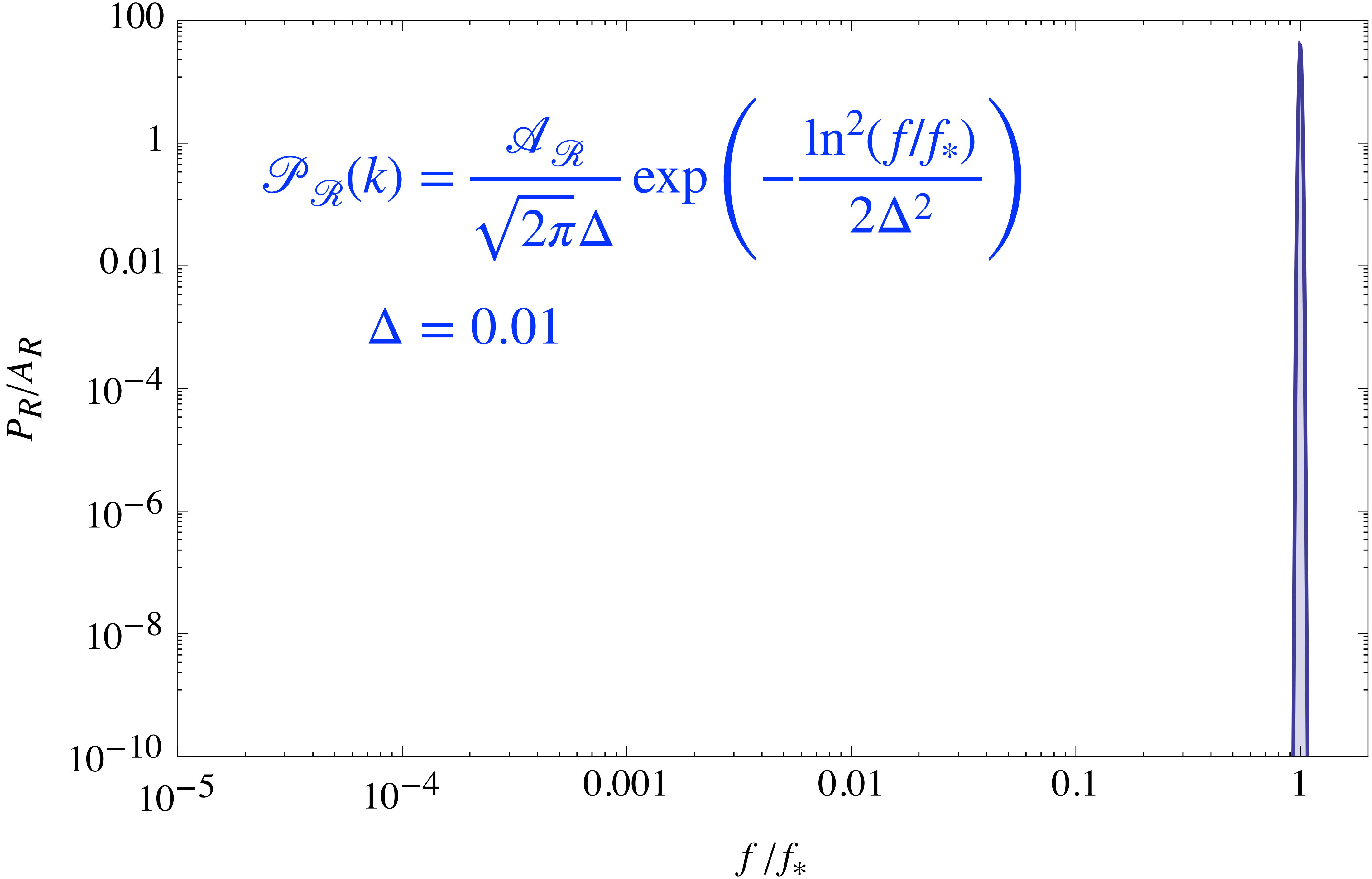
[Domènech, SP, and Sasaki, 2020]

Analytic Formula for IGW

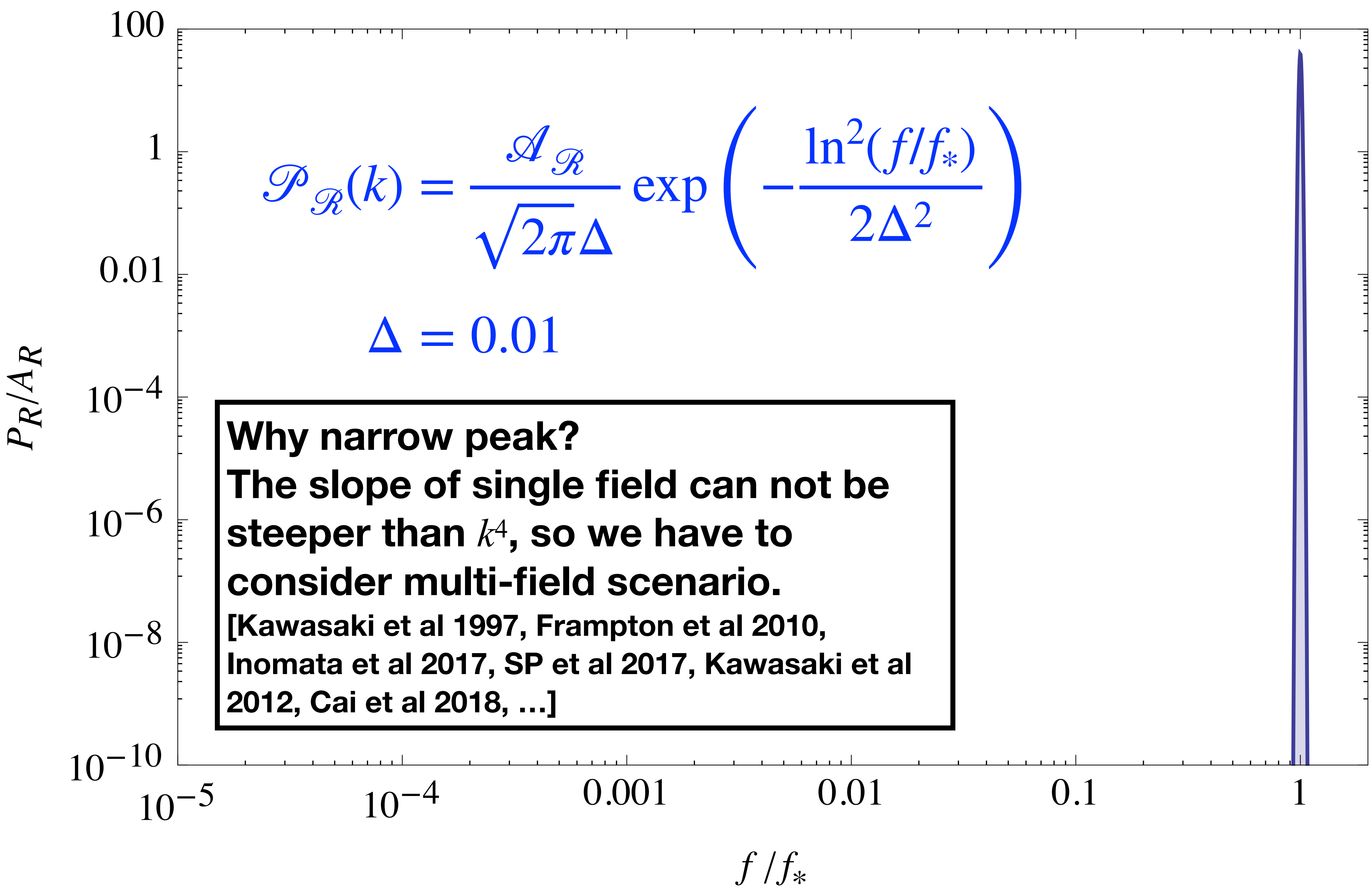
$$\Omega_{\mathrm{GW}} \equiv \frac{1}{\rho_{\mathrm{cr}}} \frac{d\rho_{\mathrm{GW}}}{d \log k}$$



Narrow Peak of $\mathscr{P}_{\mathscr{R}}$



Narrow Peak of $\mathscr{P}_{\mathscr{R}}$

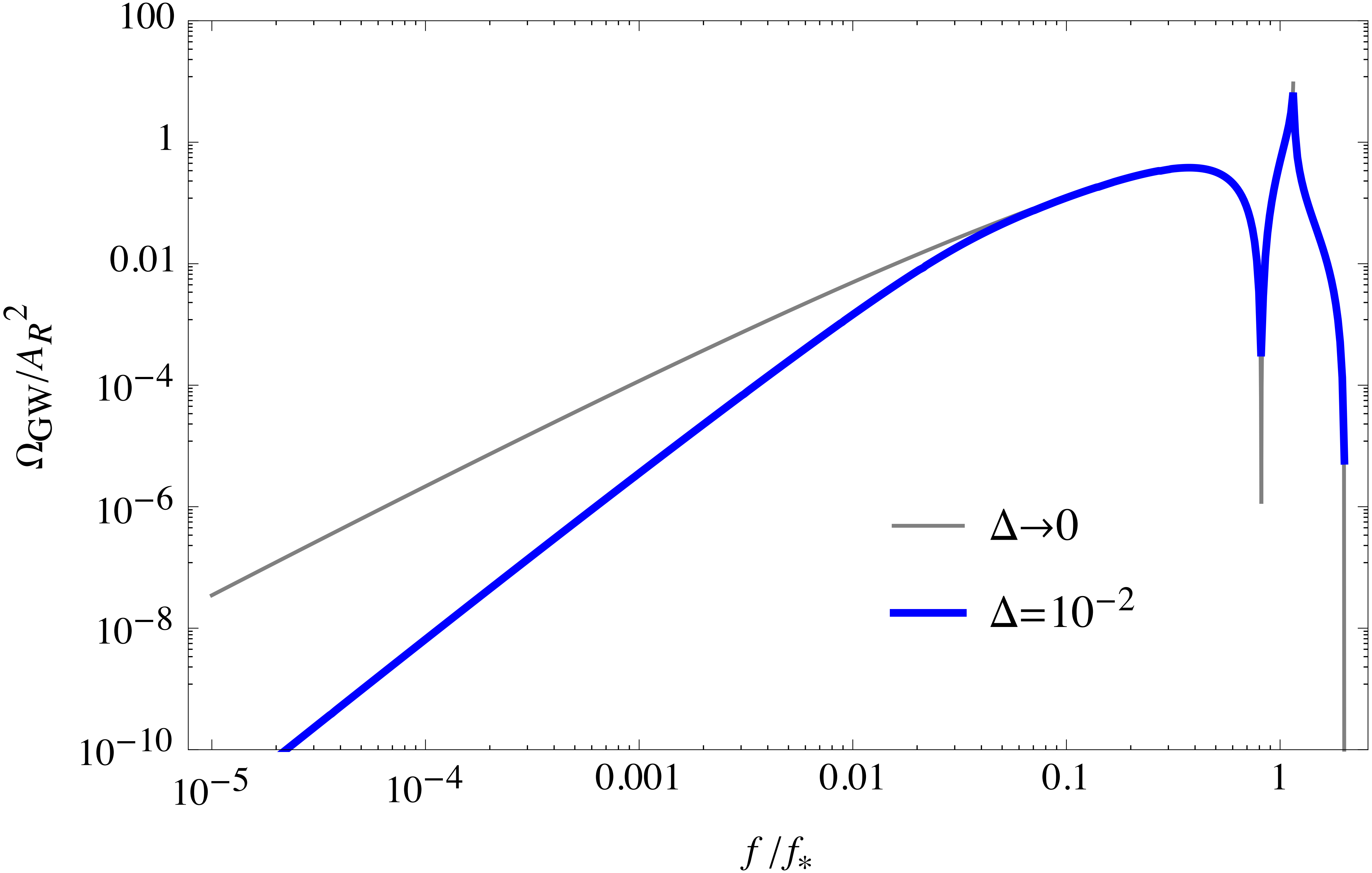


$$\mathscr{P}_{\mathscr{R}}(k) = \frac{\mathscr{A}_{\mathscr{R}}}{\sqrt{2\pi}\Delta} \exp\left(-\frac{\ln^2(f/f_*)}{2\Delta^2}\right)$$

$$\Delta = 0.01$$

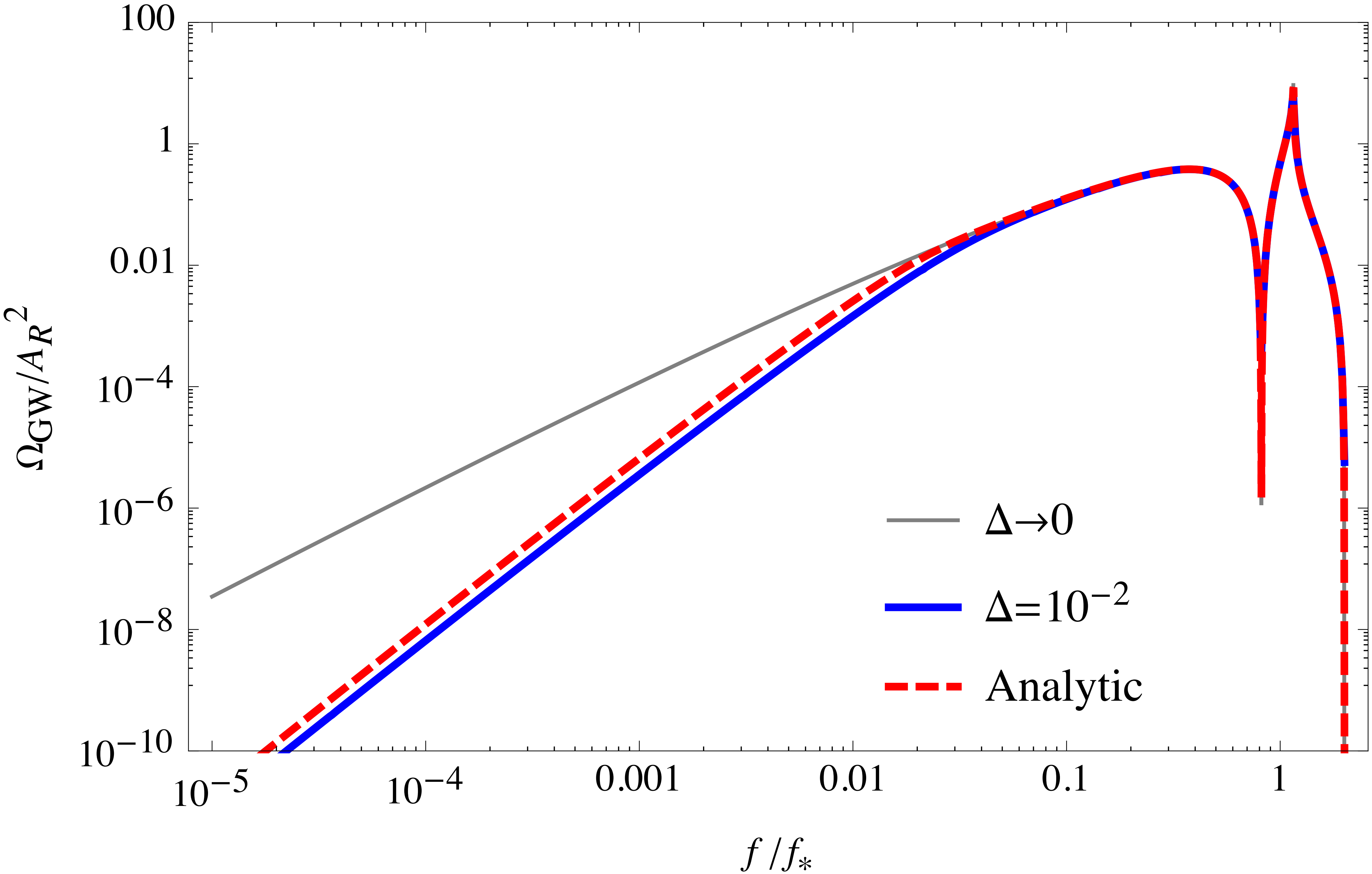
Why narrow peak?
The slope of single field can not be steeper than k^4, so we have to consider multi-field scenario.
[Kawasaki et al 1997, Frampton et al 2010, Inomata et al 2017, SP et al 2017, Kawasaki et al 2012, Cai et al 2018, …]

IGW from Narrow Peak



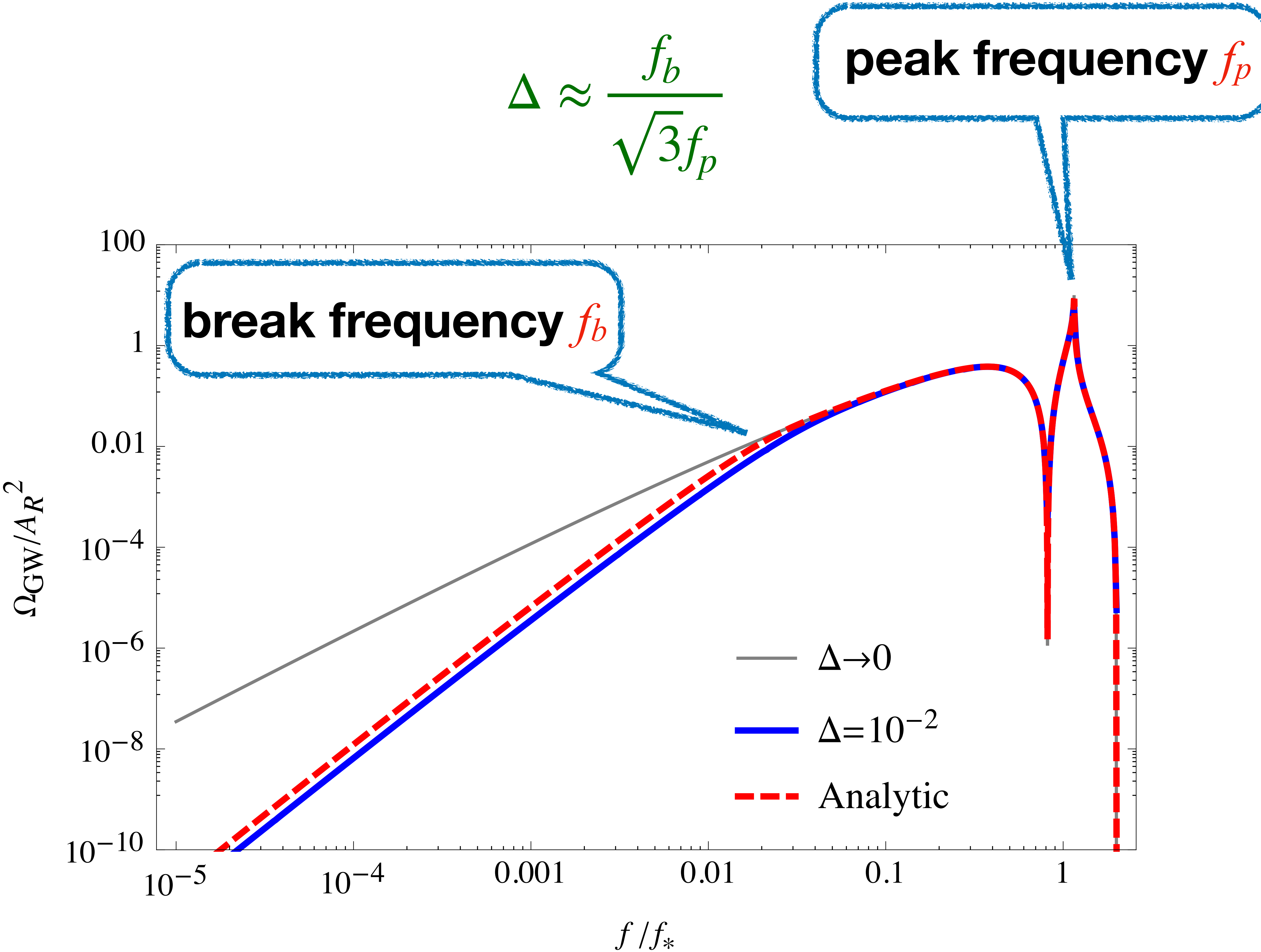
[SP and Sasaki, arXiv:2005.12306]

$$\Omega_{\rm GW}^{(\Delta\ll 1)} \approx \mathrm{erf}\left(\frac{1}{\Delta}\mathrm{arcsinh}\frac{f}{2f_*}\right)\Omega_{\rm GW}^{(\delta)}\,.$$



[SP and Sasaki, arXiv:2005.12306]

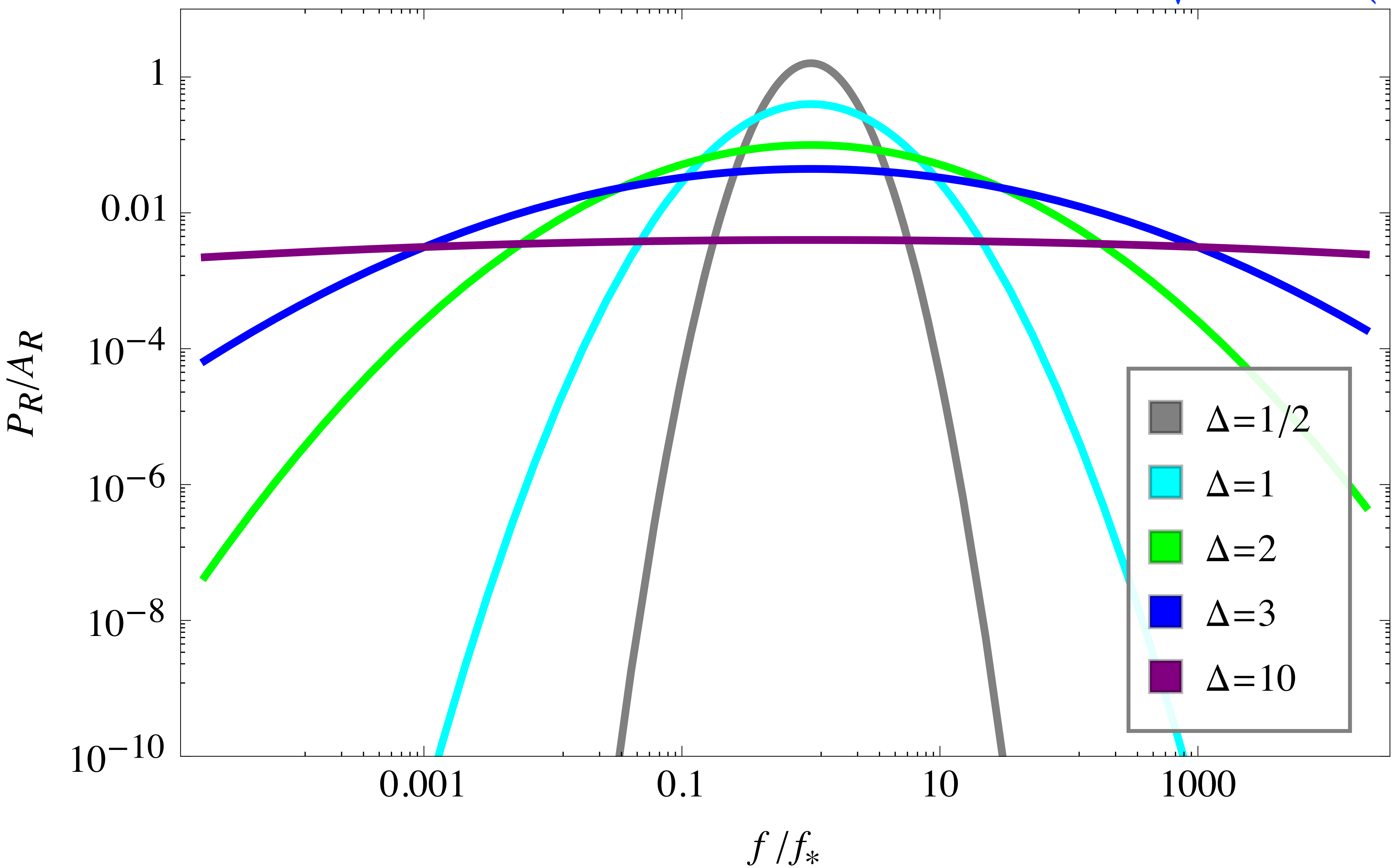
$$\Delta \approx \frac{f_b}{\sqrt{3} f_p}$$



[SP and Sasaki, arXiv:2005.12306]

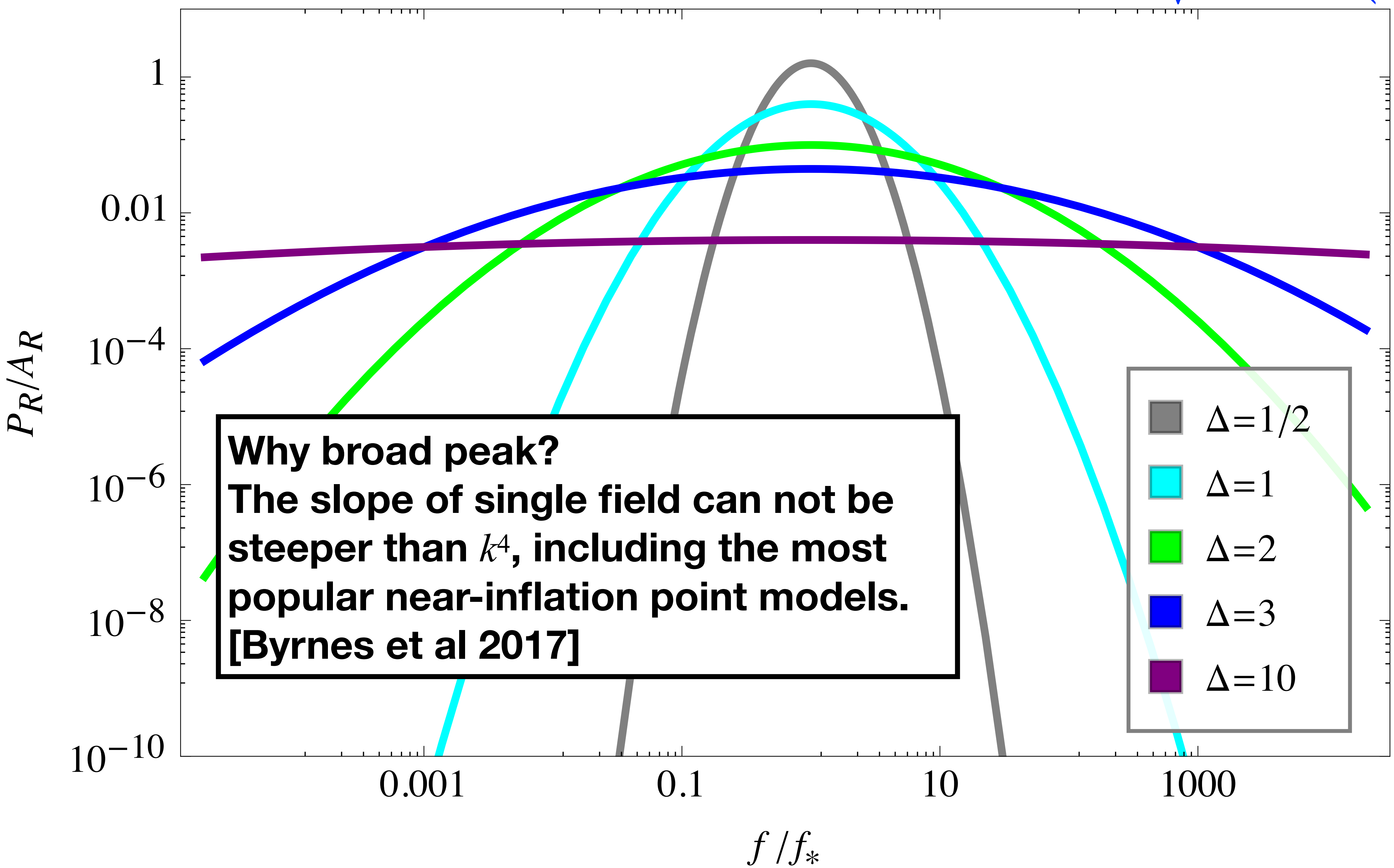
Braod Peak of $\mathscr{P}_{\mathscr{R}}$

$$\mathscr{P}_{\mathscr{R}}(k) = \frac{\mathscr{A}_{\mathscr{R}}}{\sqrt{2\pi}\Delta} \exp\left(-\frac{\ln^2(f/f_*)}{2\Delta^2}\right)$$



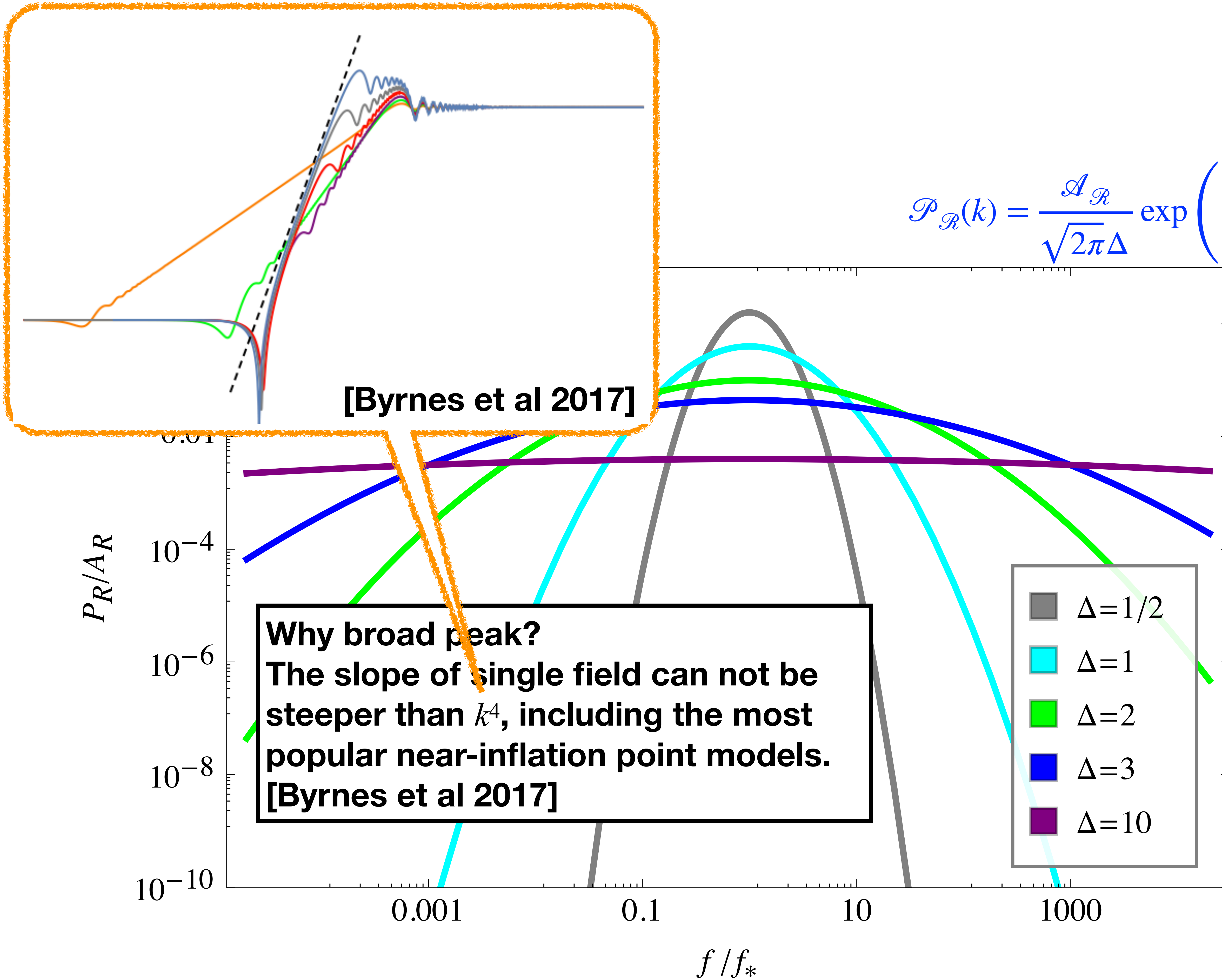
Braod Peak of $\mathscr{P}_{\mathscr{R}}$

$$\mathscr{P}_{\mathscr{R}}(k) = \frac{\mathscr{A}_{\mathscr{R}}}{\sqrt{2\pi}\Delta} \exp\left(-\frac{\ln^2(f/f_*)}{2\Delta^2}\right)$$



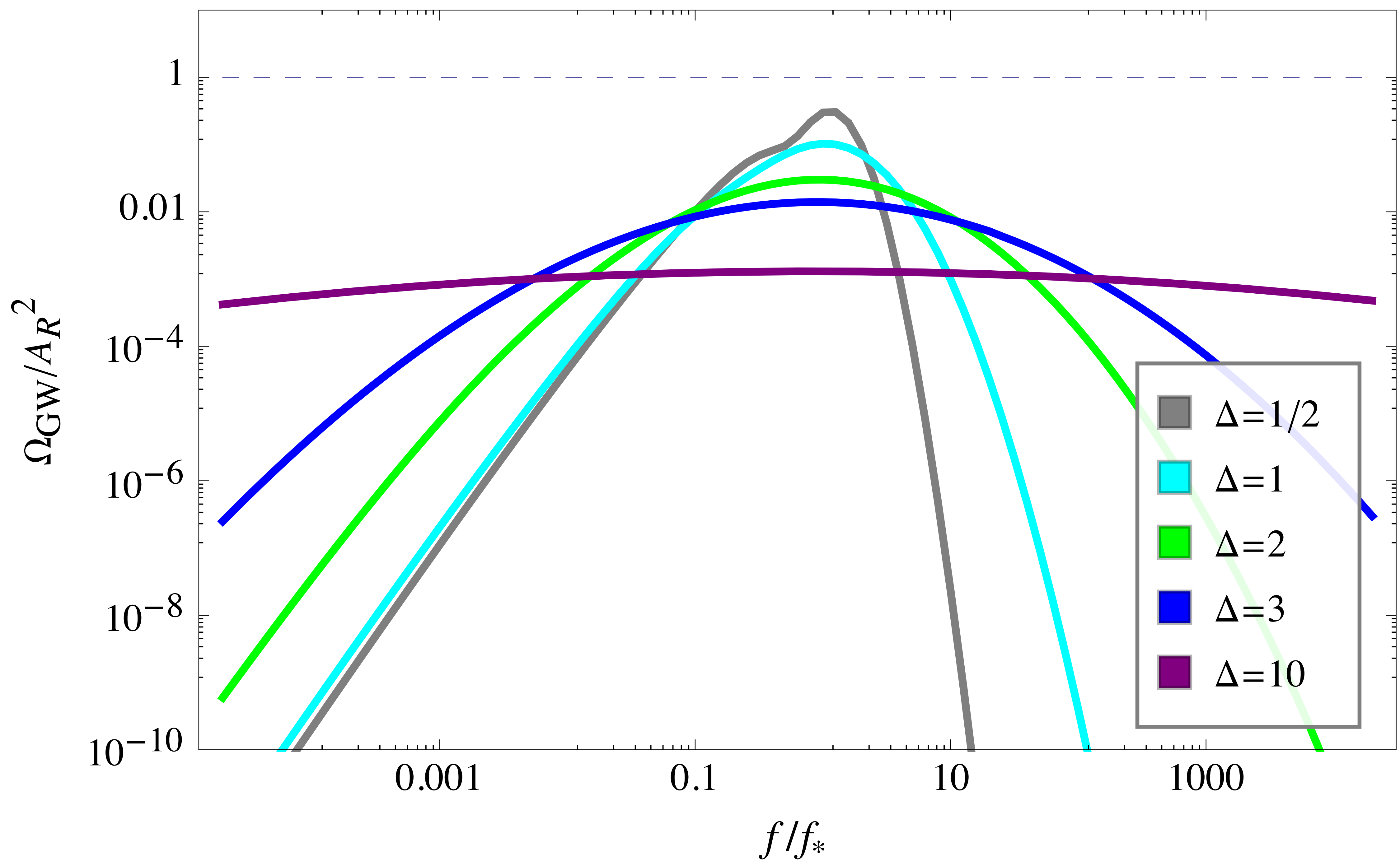
Why broad peak?
The slope of single field can not be steeper than k^4, including the most popular near-inflation point models. [Byrnes et al 2017]

$$\mathscr{P}_{\mathscr{R}}(k) = \frac{\mathscr{A}_{\mathscr{R}}}{\sqrt{2\pi}\Delta} \exp\left(-\frac{\ln^2(f/f_*)}{2\Delta^2}\right)$$



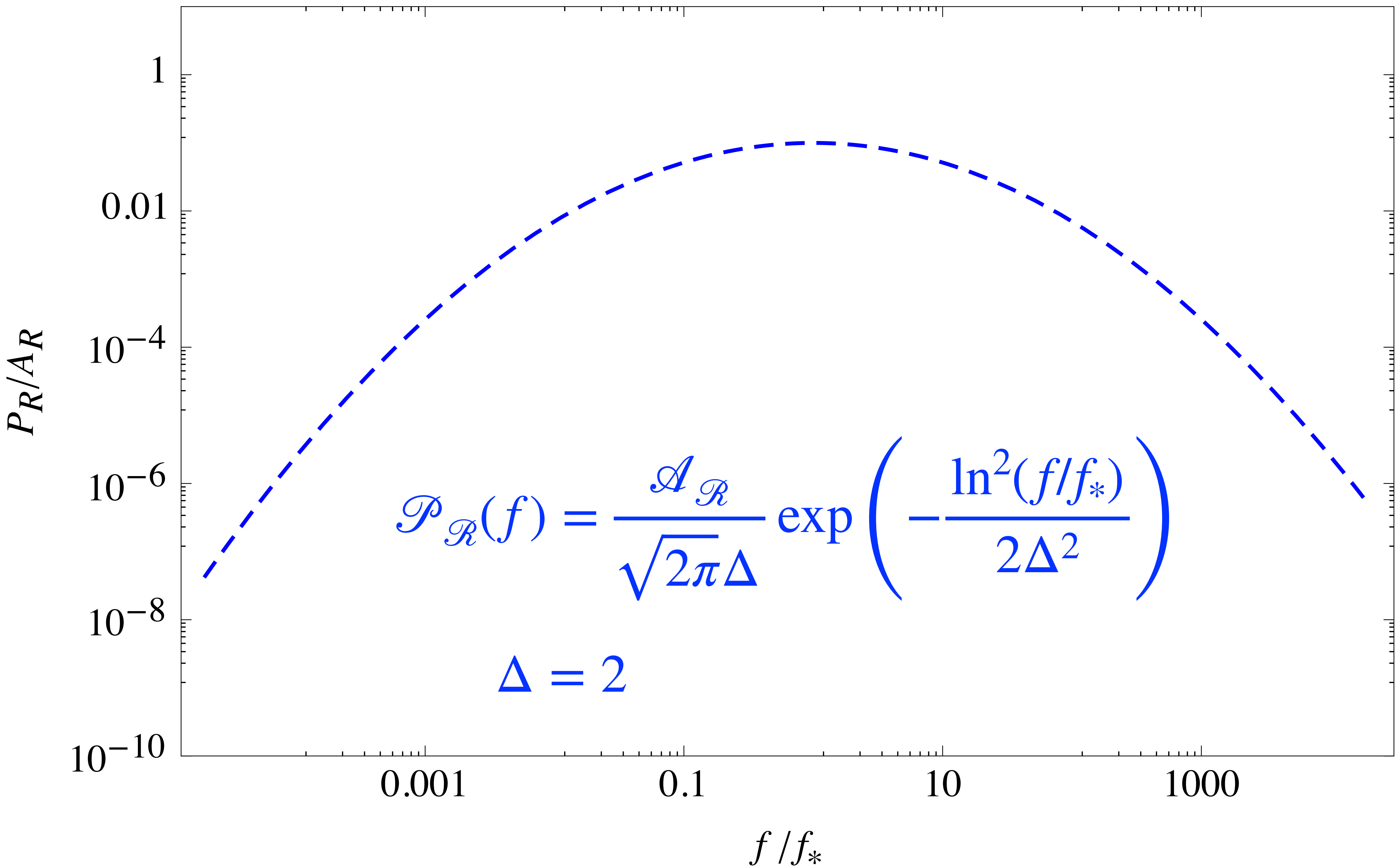
Why broad peak?
The slope of single field can not be steeper than k^4, including the most popular near-inflation point models.
[Byrnes et al 2017]

IGW from Broad Peak



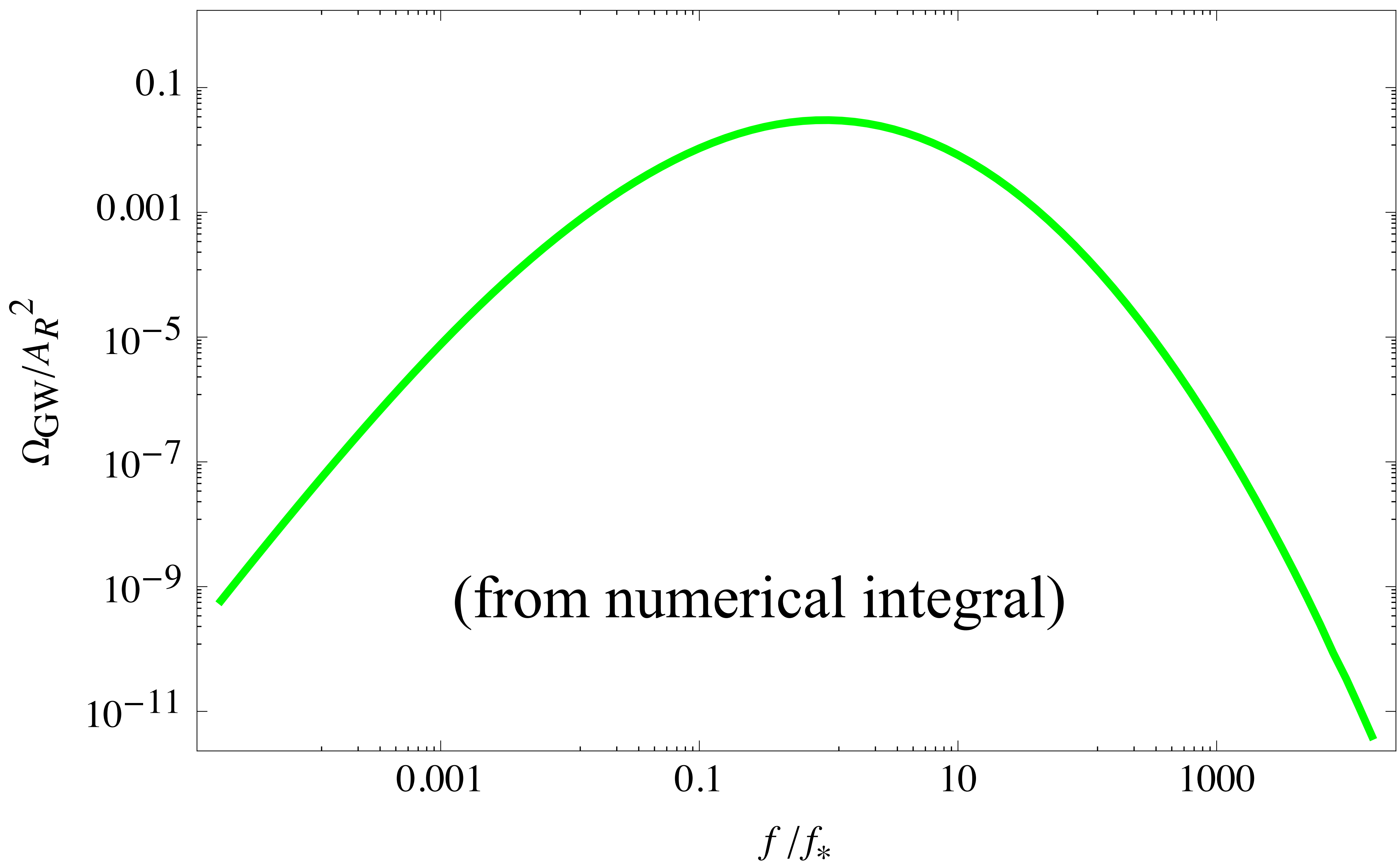
[SP and Sasaki, arXiv:2005.12306]

IGW from Broad Peak



[SP and Sasaki, arXiv:2005.12306]

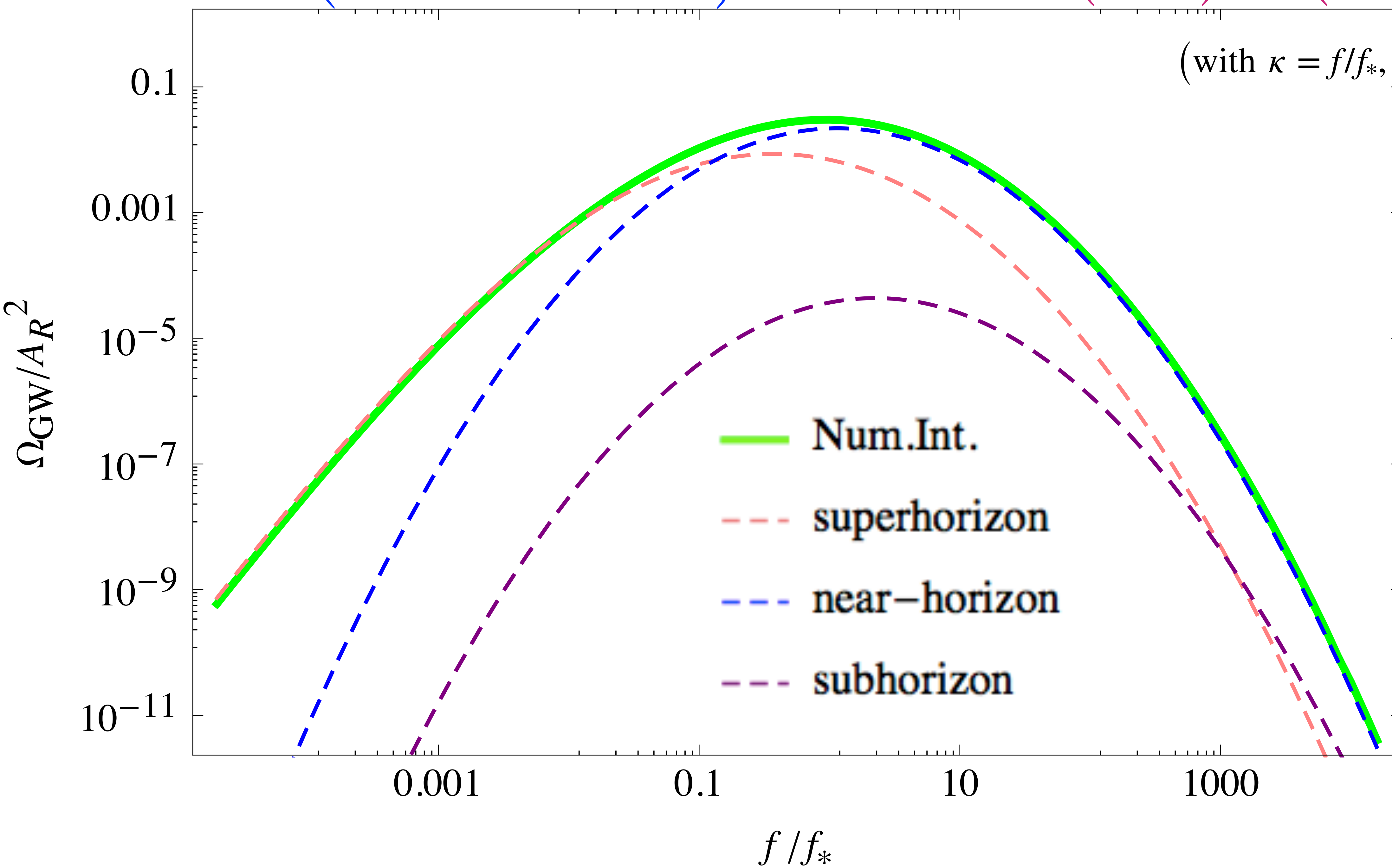
SGWB from Broad Peak



[SP and Sasaki, arXiv:2005.12306]

$$\frac{\Omega_{\rm GW}}{\mathcal{A}_{\mathcal{R}}^2} \approx \frac{4}{5\sqrt{\pi}}\kappa^3 \frac{e^{\frac{9\Delta^2}{4}}}{\Delta}\left[\left(\ln^2 K + \frac{\Delta^2}{2}\right)\mathrm{erfc}\left(\frac{\ln K + \frac{1}{2}\ln\frac{3}{2}}{\Delta}\right) - \frac{\Delta}{\sqrt{\pi}}\exp\left(-\frac{\left(\ln K + \frac{1}{2}\ln\frac{3}{2}\right)^2}{\Delta^2}\right)\left(\ln K - \frac{1}{2}\ln\frac{3}{2}\right)\right]$$
$$+\frac{0.0659}{\Delta^2}\kappa^2 e^{\Delta^2}\exp\left(-\frac{\left(\ln\kappa + \Delta^2 - \frac{1}{2}\ln\frac{4}{3}\right)^2}{\Delta^2}\right) + \frac{1}{3}\sqrt{\frac{2}{\pi}}\kappa^{-4}\frac{e^{8\Delta^2}}{\Delta}\exp\left(-\frac{\ln^2\kappa}{2\Delta^2}\right)\mathrm{erfc}\left(\frac{4\Delta^2 - \ln(\kappa/4)}{\sqrt{2}\Delta}\right)$$

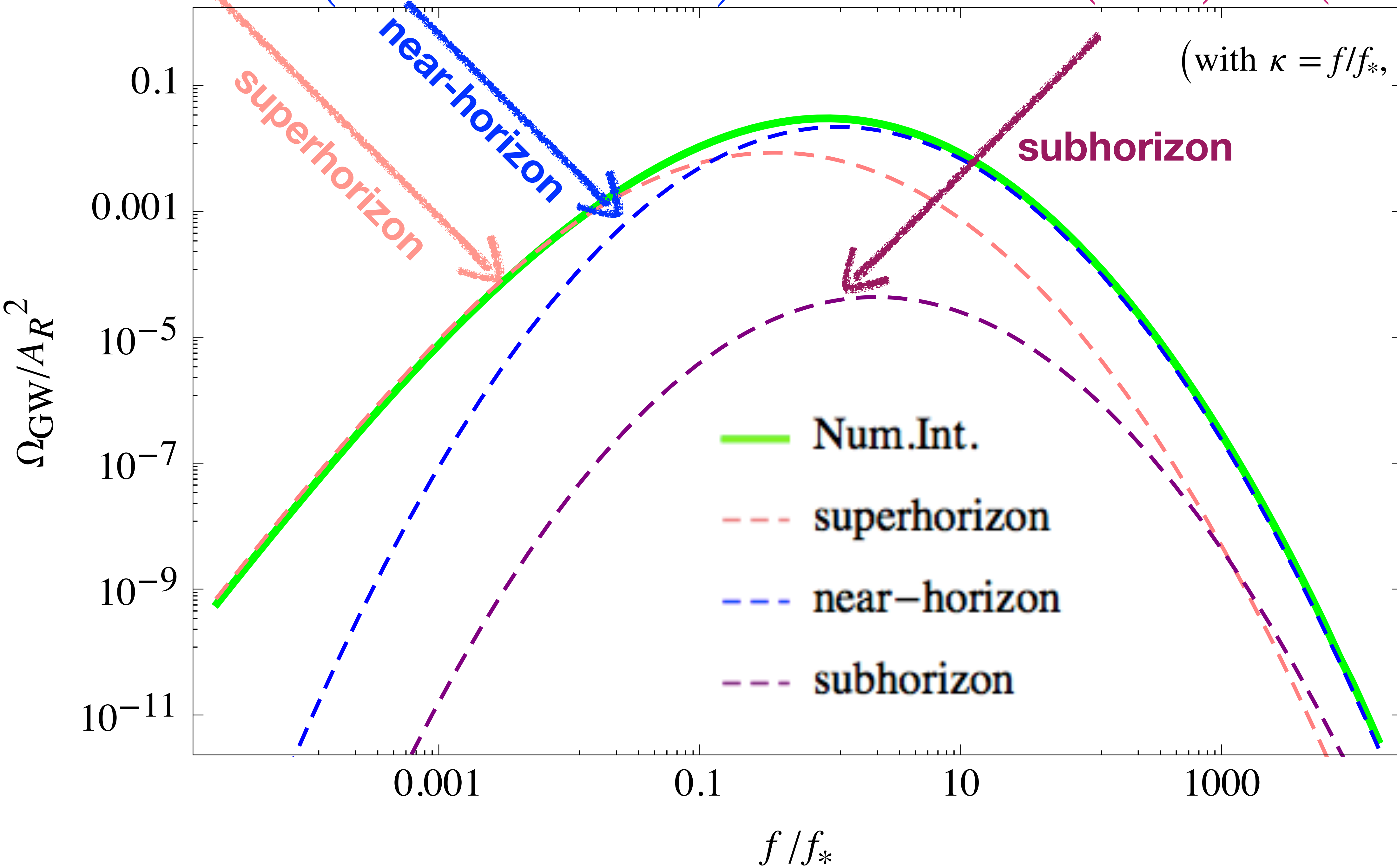
$\left(\text{with } \kappa = f/f_*,\ K = \kappa e^{3\Delta^2/2}\right)$



[SP and Sasaki, arXiv:2005.12306]

$$\frac{\Omega_{\rm GW}}{\mathcal{A}_{\mathcal{R}}^2} \approx \frac{4}{5\sqrt{\pi}}\kappa^3\frac{e^{\frac{9\Delta^2}{4}}}{\Delta}\left[\left(\ln^2 K+\frac{\Delta^2}{2}\right)\mathrm{erfc}\left(\frac{\ln K+\frac{1}{2}\ln\frac{3}{2}}{\Delta}\right)-\frac{\Delta}{\sqrt{\pi}}\exp\left(-\frac{\left(\ln K+\frac{1}{2}\ln\frac{3}{2}\right)^2}{\Delta^2}\right)\left(\ln K-\frac{1}{2}\ln\frac{3}{2}\right)\right]$$

$$+\frac{0.0659}{\Delta^2}\kappa^2 e^{\Delta^2}\exp\left(-\frac{\left(\ln\kappa+\Delta^2-\frac{1}{2}\ln\frac{4}{3}\right)^2}{\Delta^2}\right)+\frac{1}{3}\sqrt{\frac{2}{\pi}}\kappa^{-4}\frac{e^{8\Delta^2}}{\Delta}\exp\left(-\frac{\ln^2\kappa}{2\Delta^2}\right)\mathrm{erfc}\left(\frac{4\Delta^2-\ln(\kappa/4)}{\sqrt{2}\Delta}\right)$$

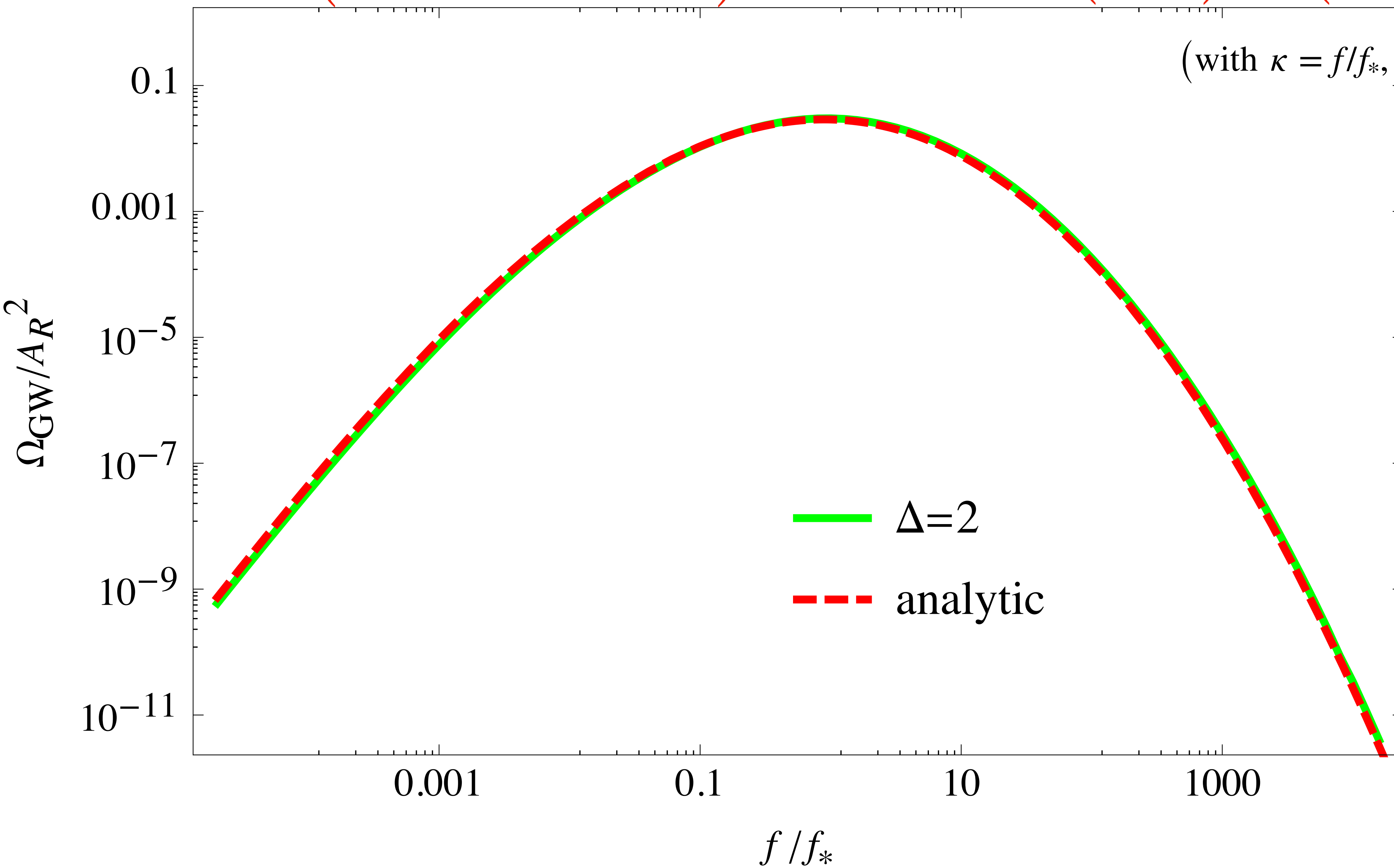


[SP and Sasaki, arXiv:2005.12306]

$$\frac{\Omega_{\rm GW}}{\mathcal{A}_{\mathcal{R}}^2} \approx \frac{4}{5\sqrt{\pi}}\kappa^3\frac{e^{\frac{9\Delta^2}{4}}}{\Delta}\left[\left(\ln^2 K + \frac{\Delta^2}{2}\right)\mathrm{erfc}\left(\frac{\ln K + \frac{1}{2}\ln\frac{3}{2}}{\Delta}\right) - \frac{\Delta}{\sqrt{\pi}}\exp\left(-\frac{\left(\ln K + \frac{1}{2}\ln\frac{3}{2}\right)^2}{\Delta^2}\right)\left(\ln K - \frac{1}{2}\ln\frac{3}{2}\right)\right]$$

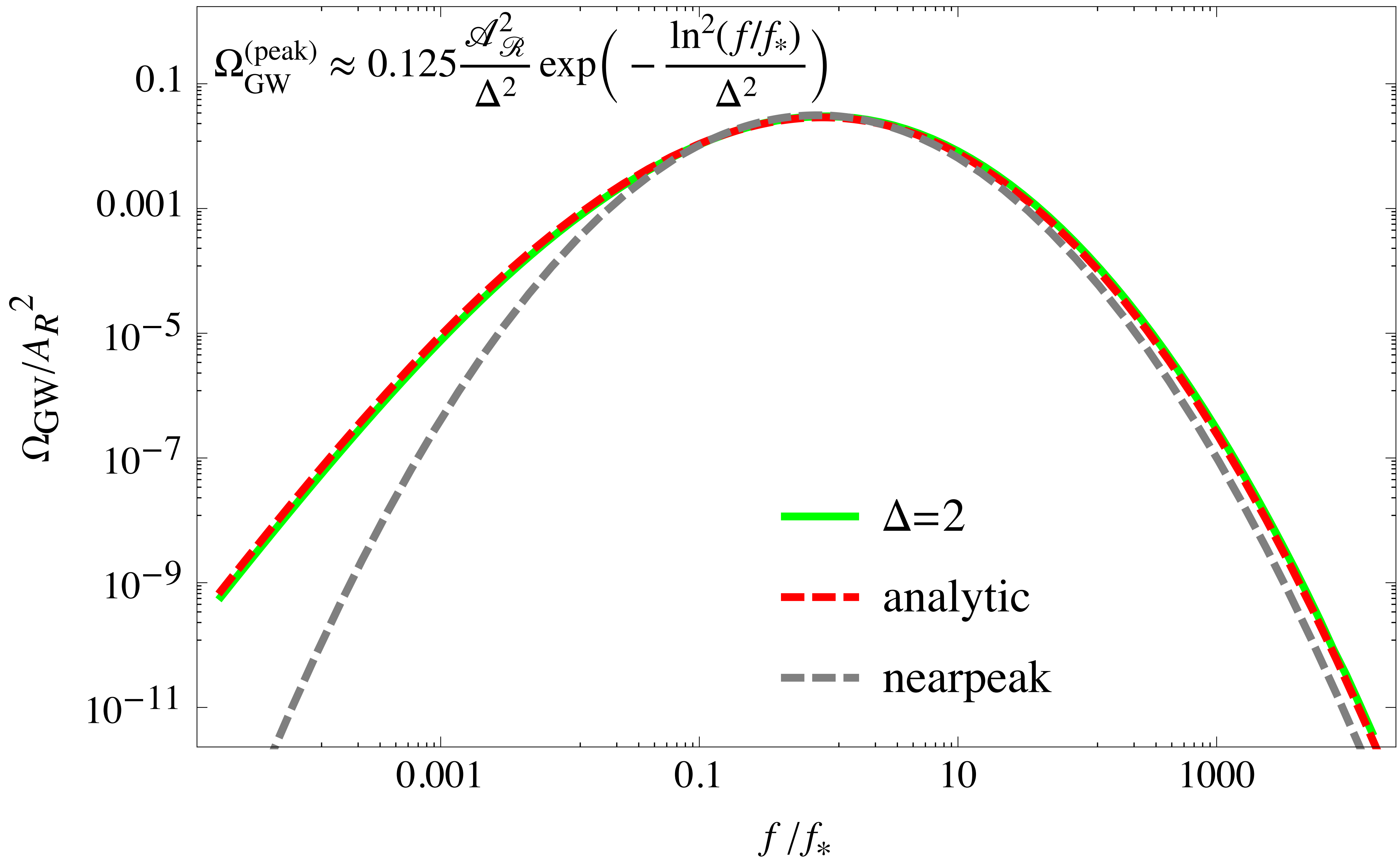
$$+\frac{0.0659}{\Delta^2}\kappa^2 e^{\Delta^2}\exp\left(-\frac{\left(\ln\kappa + \Delta^2 - \frac{1}{2}\ln\frac{4}{3}\right)^2}{\Delta^2}\right) + \frac{1}{3}\sqrt{\frac{2}{\pi}}\kappa^{-4}\frac{e^{8\Delta^2}}{\Delta}\exp\left(-\frac{\ln^2\kappa}{2\Delta^2}\right)\mathrm{erfc}\left(\frac{4\Delta^2 - \ln(\kappa/4)}{\sqrt{2}\Delta}\right)$$

$\left(\text{with } \kappa = f/f_*,\ K = \kappa e^{3\Delta^2/2}\right)$



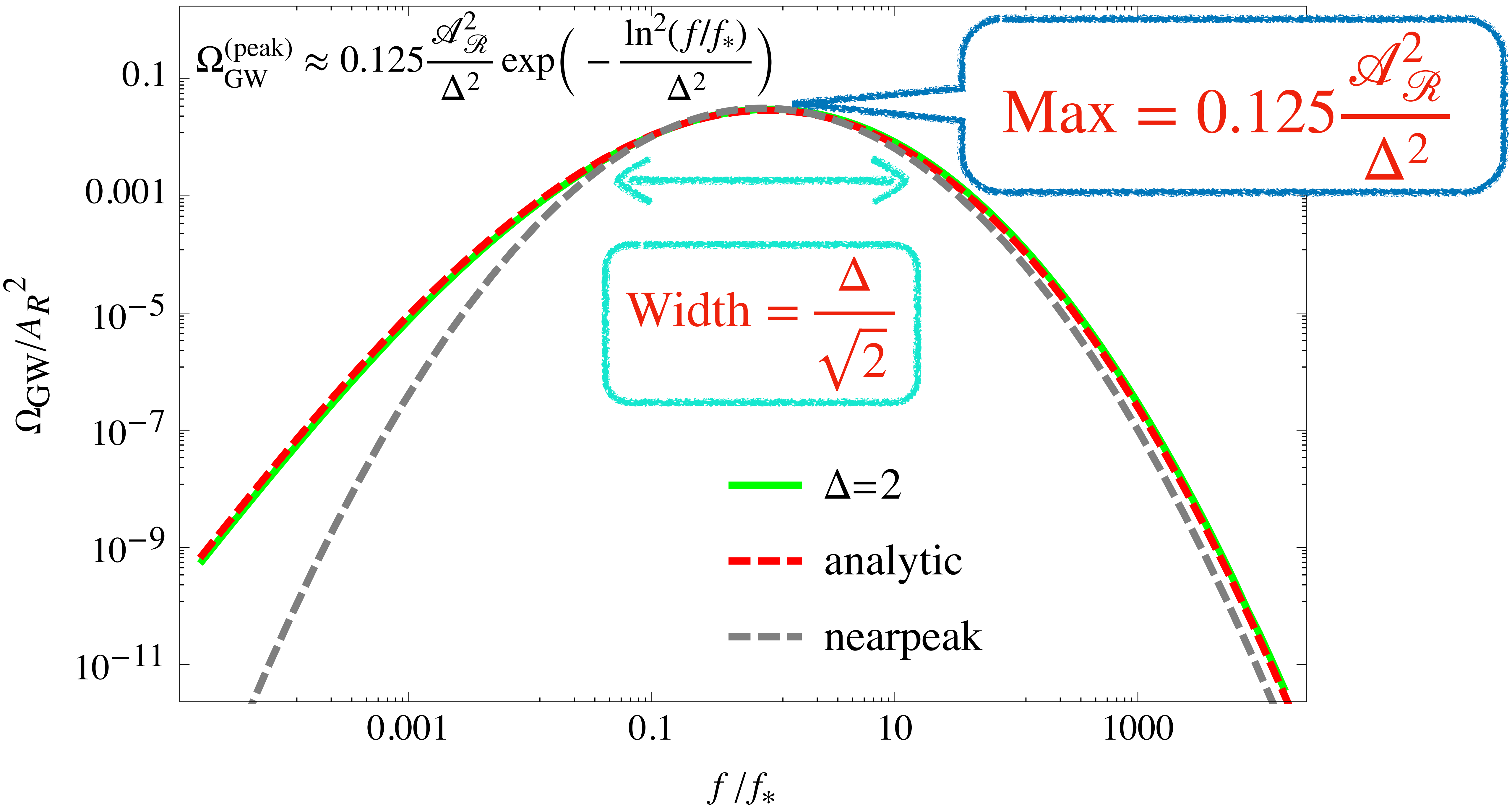
[SP and Sasaki, arXiv:2005.12306]

Take-home Result



[SP and Sasaki, arXiv:2005.12306]

Take-home Result



[SP and Sasaki, arXiv:2005.12306]

Summary

- Induced GWs is one of the most important scientific goal of the next generation GW detectors, which has fruitful phenomena together with PBH physics.
- IGW and PBH-DM: If PBHs can serve as all the DM, induced GWs must be detectable by space-based GW detectors.
- IGW and LIGO-PBH: It is in conflict with current PTA constraints, yet they are compatible if we consider non-Gaussianity.
- Shape of IGW: It depends crucially on the width of the scalar peak. We found analytical formulae for both cases in the radiation-dominated universe, which is useful for signal searching in the future.
- Infrared scaling of IGW spectrum can be used to detect the thermal history of the universe.

Thank you

for your attendance